CS 593/MA 595 - Intro to Quantum Computation
Theoretical Homework 6

Due Wednesday, October 29 at 11:59PM (upload to Brightspace)

1. Show that if A is a finite abelian group, then the dual group $\hat{A} \cong A$. [Hint: do it in two steps. First, use the fact that every finite abelian group A is a direct sum of cyclic groups to reduce to the case that $A = \mathbb{Z}/N\mathbb{Z}$. Then argue however you want that $\widehat{\mathbb{Z}/N\mathbb{Z}}$ is a cyclic group of order N.]

2. Recall that $\phi(N)$ is the number of positive integers less than N that are coprime to N. Show that if x is coprime to N and $r > 0$ is the smallest integer such that $x^r \equiv 1 \mod N$, then r divides $\phi(N)$.

3. Show directly that for any finite cyclic group $\mathbb{Z}/N\mathbb{Z}$, the Fourier basis on $\mathbb{C}^N = \mathbb{C}[\mathbb{Z}/N\mathbb{Z}]$ is an orthonormal basis.