Meeting 1.1:

0. Welcome, logistics, & surveys

I. Course overview

II. What is a manifold?

III. Why are 3-dimensional manifolds special?

Next time: Invariants of manifolds, and different encodings of 3-manifolds.

I. Course overview

Goals:

- Understand the basics of quantum computing, computational complexity, and (geometric) topology (especially knots and 3-manifolds).
- Build a precise picture of the role of topology in quantum computing, especially as a source of quantum error correcting codes and potential hardware applications via topological quantum computing.
- Develop analogies between reversible circuit models of computation and topological invariants, especially those determined by topological quantum field theories (TQFTs).
- Review the state of the art in the complexity of various topological problems

Practically, we will go backwards: topology first, then CS, then QC.

I'll lecture for several weeks, and eventually we will transition to communal learning.

II. What is a manifold?

A topological space that has a reasonable, constant notion of dimension, so every point has a neighborhood that looks like a neighborhood of a point in $\mathbb{R}^n$. More precisely:

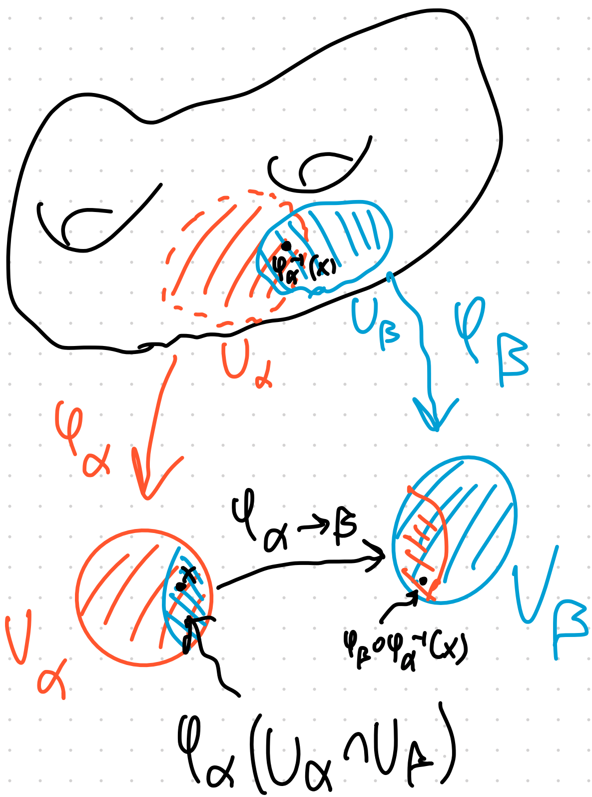
<u>Topological manifold of dimension n:</u> Topological space M^n admits an open cover $\{U_\alpha\}_{\alpha \in A}$ together w/ coordinate charts, which are homeomorphisms $\varphi_\alpha : U_\alpha \to V_\alpha$, where V_α is an ~~open set~~ open balls in $\mathbb{R}^n$.

Typically also assume M is Hausdorff and paracompact...

Problem for computability: $\mathrm{Homeo}(\mathbb{R}^n)$ is gargantuan.

Recall: the atlas of charts $\{\varphi_\alpha\}$ determines a collection of transition maps

$$\varphi_{\alpha\to\beta} : \varphi_\alpha(U_\alpha \cap U_\beta) \to \varphi_\beta(U_\alpha \cap U_\beta)$$



"Better" manifolds are formed by requiring better conditions on all of the transition maps.

E.g. the topological manifold M is equipped w/ a smooth structure if we pick an atlas of charts so that every transition map is a smooth function.

Take-away: a topological manifold might have different smooth structures.

Even smooth manifolds are too complicated to feed to a computer.

The "correct" type of manifold for inputting to computers is <u>piecewise linear.</u> After massaging the transition maps, we can feed them to a computer.

But we can go further! We can triangulate! Moreover, we get special types of triangulations.

More precisely, every PL manifold is PL homeomorphic to a simplicial complex w/ condition that the link of every vertex is a PL-sphere.

A simplicial complex is a set of vertices V together w/ a subset $C \subseteq \mathcal{P}(V)$ that is downward closed.

$V = \{a, b, c, d\}$

$C^{pre} = \{\{a, b, c\}, \{b, d\}\}$

$C = \mathcal{P}(\{a, b, c\}) \cup \mathcal{P}(\{b, d\})$