Exam 1 Review

15 questions multiple choice - scantron

ES 2107 Room @ 8pm-9pm on Monday

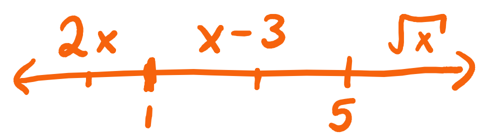
1f Given $h(x) = 3x + 5$, find and simplify $h(2a) + h(a+1)$

$h(2a) = 3(2a) + 5 = 6a + 5$

$h(a+1) = 3(a+1) + 5 = 3a + 3 + 5 = 3a + 8$

$h(2a) + h(a+1) = 6a + 5 + 3a + 8 = 9a + 13$

1h Given $g(x) = \begin{cases} 2x, & x < 1 \\ x - 3, & 1 \le x < 5 \\ \sqrt{x}, & x \ge 5 \end{cases}$



Find $g(0)$, $g(1)$, $g(4)$, $g(9)$

$g(0) = 2(0) = 0$

$g(1) = (1) - 3 = -2$

$g(4) = (4) - 3 = 1$

$g(9) = \sqrt{9} = 3$

2c Given $f(x) = 4 - x$ and $g(x) = x + 2$, find and simplify

(iv) $(f \cdot g)(x) \stackrel{def}{=} (f(x))(g(x))$

$= (4 - x)(x + 2)$

$= 4x - x^2 + 8 - 2x$

$= -x^2 + 2x + 8$

	x	2
4	$4x$	8
$-x$	$-x^2$	$-2x$

$= -x^2 + 2x + 8$

(3a) Given $f(x) = \frac{1}{x}$ and $g(x) = x - 3$, find and simplify

(iii) $(f \circ f)(x) \stackrel{def}{=} f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = 1 \cdot \frac{x}{1} = x$

(iv) $(g \circ g)(x) \stackrel{def}{=} g(g(x)) = g(x-3) = (x-3) - 3 = x - 6$

(3g) Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-4}$, find the

(a) Domain of $(f \circ g)(x)$.

↳ Domain of $g(x)$ AND domain of $(f \circ g)(x)$.

$$(f \circ g)(x) \stackrel{def}{=} f(g(x)) = f\left(\frac{1}{x-4}\right) = \sqrt{\frac{1}{x-4}} = \frac{\sqrt{1}}{\sqrt{x-4}} = \frac{1}{\sqrt{x-4}}$$

Domain of $g(x)$ is

$\frac{1}{x-4} \Rightarrow x - 4 \neq 0 \Rightarrow x \neq 4$

Domain of $(f \circ g)(x)$ is

$\frac{1}{\sqrt{x-4}} \Rightarrow x - 4 > 0 \Rightarrow x > 4$

When does both of these happen?

$x > 4$

(3g) Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-4}$, find the

(b) $(g \circ f)(x)$ Domain of it

↳ Domain of $f(x)$ and Domain of $(g \circ f)(x)$

$$(g \circ f)(x) \stackrel{def}{=} g(f(x)) = g(\sqrt{x}) = \frac{1}{\sqrt{x} - 4}$$

Domain of $f(x)$ is

$\sqrt{x} \Rightarrow x \geq 0$

When does $x \geq 0$, $x \geq 0$, $x \neq 16$ happen?

Domain of $f(x)$ is

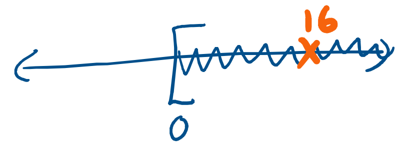
$\sqrt{x} \Rightarrow x \geq 0$

Domain of $(g \circ f)(x)$ is

- $\sqrt{x} \Rightarrow x \geq 0$
- $\dfrac{1}{\sqrt{x}-4} \Rightarrow \sqrt{x}-4 \neq 0$

$$\sqrt{x} \neq 4$$
$$(\sqrt{x})^2 \neq 4^2$$
$$x \neq 16$$

happen?



$[0, 16) \cup (16, \infty)$

④ You know

$$\frac{x^m}{x^n} = x^{m-n} \qquad x^m \cdot x^n = x^{m+n} \qquad (x^m)^n = x^{m \cdot n} \qquad x^{-m} = \frac{1}{x^m}$$

ⓐ Simplify the expression

$$\sqrt{\frac{18xy^5}{x^4}} = \sqrt{\frac{18y^5}{x^3}} = \frac{\sqrt{18} \cdot \sqrt{y^5}}{\sqrt{x^3}} = \frac{\sqrt{2} \cdot \cancel{\sqrt{9}}^{\,3} \cdot \sqrt{y} \cdot \cancel{\sqrt{y^4}}^{\,y^2}}{\sqrt{x} \cdot \cancel{\sqrt{x^2}}^{\,x}}$$

$$= \frac{\sqrt{2} \cdot \sqrt{y} \cdot 3y^2}{\sqrt{x} \cdot x} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{3y^2 \sqrt{2} \cdot \sqrt{y} \cdot \sqrt{x}}{x \cdot x}$$

$$= \frac{3y^2 \sqrt{2xy}}{x^2}$$

(5e) Rationalize the denominator in the expression

$$\frac{1}{\sqrt{x+3} - \sqrt{3}}$$

$$= \frac{1}{\sqrt{x+3} - \sqrt{3}} \cdot \frac{\sqrt{x+3} + \sqrt{3}}{\sqrt{x+3} + \sqrt{3}}$$

$(a-b)(a+b) = a^2 - b^2$

Conjugates

$a-b \Rightarrow a+b$

$a+b \Rightarrow a-b$

$$= \frac{\quad}{\sqrt{x+3}-\sqrt{3}} \cdot \frac{\sqrt{x+3}+\sqrt{3}}{\sqrt{x+3}+\sqrt{3}}$$

$a+b \Rightarrow a-b$

$$= \frac{\sqrt{x+3}+\sqrt{3}}{(\sqrt{x+3})^2-(\sqrt{3})^2} = \frac{\sqrt{x+3}+\sqrt{3}}{x+3-3} = \frac{\sqrt{x+3}+\sqrt{3}}{x}$$

5a Simplify the expression

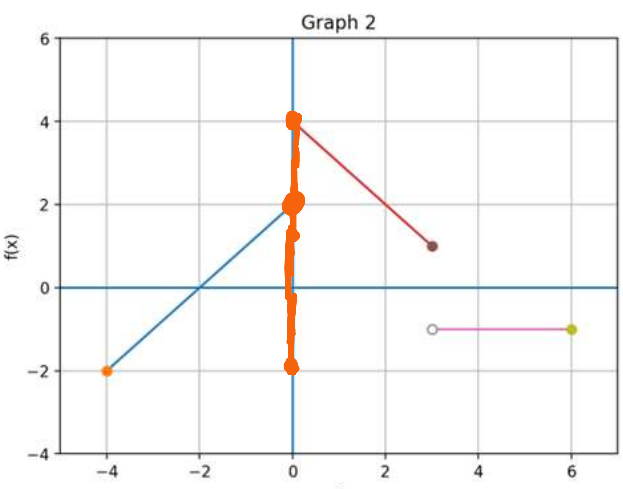
$$\frac{\frac{1}{2x}}{1+\frac{1}{2x+1}}$$

Combine the denominator and then do "keep change flip"

$$= \frac{\frac{1}{2x}}{\frac{2x+1}{2x+1}+\frac{1}{2x+1}} = \frac{\frac{1}{2x}}{\frac{2x+1+1}{2x+1}} = \frac{\frac{1}{2x}}{\frac{2x+2}{2x+1}} = \frac{1}{2x} \cdot \frac{2x+1}{2x+2} = \frac{1}{2x} \cdot \frac{2x+1}{2(x+1)}$$

$$= \frac{2x+1}{4x(x+1)}$$

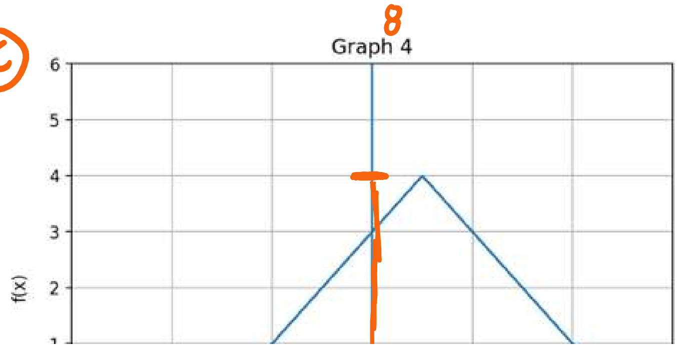
9a Range



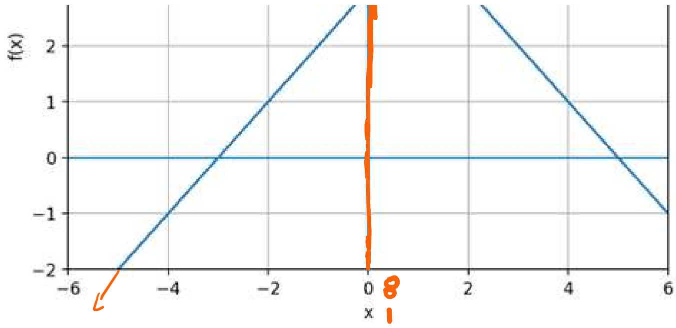
$[-2, 4]$

c

8



Range is $(-\infty, 4]$



7a) Given $f(x) = \underset{a}{2}x^2 - \underset{b}{9}x - \underset{c}{5}$ solve $f(x) = 0$.

$a \cdot c = 2 \cdot (-5) = -10$

$1 \cdot -10 = -9$

$2 \cdot 5$

$$2x^2 - 9x - 5$$
$$= 2x^2 + x - 10x - 5$$
$$= x(2x+1) - 5(2x+1)$$
$$= (x-5)(2x+1) = 0$$

$x - 5 = 0$ $\quad$ $2x + 1 = 0$

$x = 5$ $\quad$ $x = -\frac{1}{2}$

$\Rightarrow x = -\frac{1}{2}, 5$