Exam 2 on Wednesday this week March 4
@ 8pm-9pm in ES2107

(2a) The graph has a pt $(4,-3)$. What is the pt on $y=f(x+5)$?

Left by 5 $\Leftrightarrow$ i.e. subtracting x by 5

$(4-5, -3)$

$\boxed{(-1,-3)}$

(2b) The graph has a pt $(-2,6)$. What is the pt on $y=f(x)+4$?

$(-2, 6+4)$

$\boxed{(-2,10)}$

(3) Discriminant is b^2-4ac

$b^2-4ac>0 \Rightarrow$ 2 real solutions

$b^2-4ac=0 \Rightarrow$ 1 real solution

$b^2-4ac<0 \Rightarrow$ No real solutions (complex)

(a) $f(x)=x^2-5x+6$

$b^2-4ac = (-5)^2-4(1)(6)$

$= 25-24$

$=1 \quad >0 \Rightarrow$ 2 real. .

$= 25 - 24$
$= 1 \quad > 0 \Rightarrow$ 2 real solutions

④ Solving quadratic equations do whatever way (i.e. factoring or quadratic formula)

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

ⓐ $x^2 - 12x + 36 = (x)^2 - \underset{2x \cdot 6\,?}{12x} + 6^2$

$= (x-6)^2 = 0$

$x = 6$

⑤ Quadratic Eqns to Vertex Form (completing the square)

ⓐ $f(x) = \underbrace{2x^2 + 8x}\, + 6$

$= 2(x^2 + 4x) + 6$

$\downarrow$ $\div 2$ and then square it

$= 2(x^2 + 4x + 2^2 - 2^2) + 6$

$= 2(x^2 + 4x + 2^2) - 2 \cdot 2^2 + 6$

$= 2(x+2)^2 - 8 + 6$

$= 2(x+2)^2 - 2$

ⓓ $f(x) = \underbrace{4x^2 - 4x}\, + 9$

(d) $f(x) = 4x^2 - 4x + 9$

$= 4(x^2 - 1x) + 9$

$\div 2$ and square it

$= 4\left(x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right) + 9$

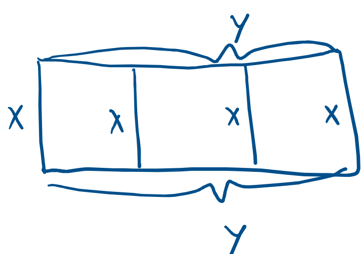
$= 4\left(x^2 - x + \left(\frac{1}{2}\right)^2\right) - 4\left(\frac{1}{2}\right)^2 + 9$

$= 4\left(x - \frac{1}{2}\right)^2 - 4\left(\frac{1}{4}\right) + 9$

$= 4\left(x - \frac{1}{2}\right)^2 - 1 + 9$

$= 4\left(x - \frac{1}{2}\right)^2 + 8$

(6a)



$P = 640 = 4x + 2y \iff 320 = 2x + y$

$A = xy \leftarrow y = 320 - 2x$

$A = x(320 - 2x)$

$= -2x^2 + 320x$

$$\begin{array}{r} 6400 \\ \times \quad 2 \\ \hline 12800 \end{array}$$

Next complete the square.

$A = -2(x^2 - 160x)$

$\div 2$ and square it

$= -2\left(x^2 - 160x + (80)^2 - (80)^2\right)$

$= -2\left(x^2 - 160x + (80)^2\right) - (-2) \cdot 80^2$

$= -2(x - 80)^2 + 12800$

Vertex $(80, 12800)$

$\downarrow$

Max Area

$a(x-h)^2 + k$

Max Area

7. Just plug in $x=0$ to find the y-intercept.

8a. Zeroes of $p(x) = x^3 - 3x^2 - 4x + 12$ that has a factor $x-2$

$\Downarrow$

$x = 2 \Rightarrow$ zero of $p(x)$

$$\begin{array}{c|cccc} 2 & 1 & -3 & -4 & 12 \\ & \downarrow & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$$p(x) = (x-2)(x^2 - x - 6)$$
$$= (x-2)(x-3)(x+2) = 0$$
$$x = 2, 3, -2$$

9. Vertical Asymptote vs. Hole.

Simplify the rational function before determining anything.

1. If something cancels $\Rightarrow$ Hole
2. If there is a factor in the denominator $\Rightarrow$ VA.

i) $f(x) = \dfrac{x^2 - 9}{x^2 + 2x - 15} = \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+5)}$

Hole @ $x - 3 = 0$
$x = 3$

VA: $x + 5 = 0$
$x = -5$

Horizontal:

(1) If deg(numerator) < deg(denominator),
then HA is $y=0$.

(f) $f(x) = \dfrac{6x-1}{x^2+4} \Rightarrow$ HA @ $y=0$

(2) If deg(numerator) = deg(denominator),
then HA is $y = \dfrac{(\text{leading term of numerator})}{(\text{leading term of denominator})}$

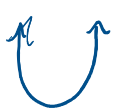
(b) $f(x) = \dfrac{3x^2-4}{x^2+1} \Rightarrow$ HA @ $y = \dfrac{3x^2}{x^2} = 3$

(3) If deg(numerator) > deg(denominator),
then **NO HA!** (Really a slant)

(d) $f(x) = \dfrac{5x^3+2x}{x^2-1} \Rightarrow$ No HA

FYI No Slant Asymptotes on Exam 2

(10) End behavior:

 + Even leading power $\Rightarrow$ $x \to -\infty$ $f(x) \to \infty$; $x \to +\infty$ $f(x) \to \infty$

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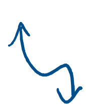 + odd leading power $\Rightarrow$ $x \to -\infty$ $f(x) \to \infty$; $x \to +\infty$ $f(x) \to -\infty$

$\updownarrow$ + odd leading power $\Rightarrow$ $x \to -\infty$ $f(x) \to \infty$

$x \to +\infty$ $f(x) \to -\infty$

ⓐ $f(x) = 3x(x-4)^2(x+1)$

$3x \cdot x^2 \cdot x = 3x^4$ >0, even

$x \to -\infty$ $f(x) \to \infty$

$x \to +\infty$ $f(x) \to \infty$

(11) Problem 1: ~~A~~ ~~B~~ C ~~D~~ E

Zeroes $\Rightarrow$ Factors in numerators

✓ (i) Zeros @ $x=-6$ and $x=2$

$\Rightarrow$ Numerator $(x+6)(x-2)$

Pin (ii) HA: @ $y=-1$ so leading terms match

✓ (iii) VA @ $x=4$ $\Rightarrow$ Denominator $x-4$

✓ (iv) Hole at $x=-3$ $\Rightarrow$ Numerator + Denominator have $x+3$

(12) To determine the function using a graph remember

(1) Holes are when factors cancel

(2) VA are factors that remain in the denominator.

ⓑ Note VA @ $x=-4$ $\Leftrightarrow$ $x+4$ is in the denominator

Note Hole @ $x=1$ $\Leftrightarrow$ $x-1$ is in both numerator and denominator

i.e. $f(x) = \dfrac{x-1}{(x-1)(x+4)}$

But that logic E is our answer.