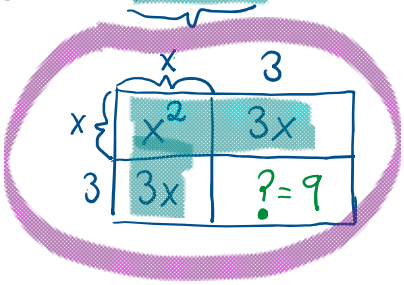
No Class Friday 2/13 ⇒ No written homework next week

Lesson 10: Intro to Quadratic Functions

Completing the Square

- Quadratic function: $f(x) = ax^2 + bx + c$
- We want to find solutions for $f(x) = 0$
  i.e. $ax^2 + bx + c = 0$
- We have seen factoring, now we will complete the square.
- Ex: $x^2 + 6x = 10$



Want is $(\quad)^2$ for all of it?

$x^2 + 6x = (\quad)^2$

if I add 9 and subtract it.

$x^2 + 6x + 9 - 9$

$= (x+3)^2 - 9$

Solve $x^2 + 6x = 10$

By green stuff, rewrite the LHS

$(x+3)^2 - 9 = 10$

$(x+3)^2 = 19$

$x + 3 = \pm\sqrt{19}$

$x = -3 \pm \sqrt{19}$

Note: $y^2 = d$ when $d \geq 0$ ⇒ $y = \pm\sqrt{d}$

In general $\left(x + \frac{b}{2}\right)^2 = x^2 + bx + \left(\frac{b}{2}\right)^2$

Solve the following quadratic eqns by completing the square:

Ex 1: Solve the following quadratic eqns by completing the square:

$(x+a)^2 = x^2 + 2xa + a^2$

(a) $x^2 + 16x + 7 = 0$

$x^2 + 16x = -7$

Complete the square

So find $\frac{b}{2} = \frac{16}{2} = 8$

So add $\left(\frac{b}{2}\right)^2 = 8^2$ to both sides

$x^2 + 16x + 8^2 = -7 + 8^2$

$(x+8)^2 = -7 + 64 = 57$

$x + 8 = \pm\sqrt{57}$

$x = -8 \pm \sqrt{57}$

(b) $x^2 + \frac{5}{6}x - \frac{1}{6} = 0$

$x^2 + \frac{5}{6}x = \frac{1}{6}$

Complete the square

So find $\frac{b}{2} = \frac{5/6}{2} = \frac{5}{6} \cdot \frac{1}{2} = \frac{5}{12}$

So add $\left(\frac{b}{2}\right)^2 = \left(\frac{5}{12}\right)^2$ to both sides

$x^2 + \frac{5}{6}x + \left(\frac{5}{12}\right)^2 = \frac{1}{6} + \left(\frac{5}{12}\right)^2$

$\left(x + \frac{5}{12}\right)^2 = \frac{1}{6} + \frac{25}{144}$

$\left(x + \frac{5}{12}\right)^2 = \frac{1}{6} \cdot \frac{24}{24} + \frac{25}{144}$

$\left(x + \frac{5}{12}\right)^2 = \frac{24}{144} + \frac{25}{144}$

$\left(x + \frac{5}{12}\right)^2 = \frac{49}{144}$

$x + \frac{5}{12} = \pm\sqrt{\frac{49}{144}} = \pm\frac{7}{12}$

$$x + \frac{5}{12} = \pm\sqrt{\frac{49}{144}} = \pm\frac{7}{12}$$

$$x = \frac{-5}{12} \pm \frac{7}{12}$$

$$x = -\frac{5}{12} + \frac{7}{12} = \frac{2}{12} = 1/6 \qquad x = \frac{-5}{12} - \frac{7}{12} = \frac{-12}{12} = -1$$

(c) $2x^2 - 8x - 5 = 0$

$2x^2 - 8x = 5$

$2(x^2 - 4x) = 5$

Factor b/c I need $x^2 \pm bx$

$x^2 - 4x = \frac{5}{2}$

Complete the square

So find $\frac{b}{2} = \frac{-4}{2} = -2$

Add to both sides $(-2)^2$

$$x^2 - 4x + (-2)^2 = \frac{5}{2} + (-2)^2$$

$$(x-2)^2 = \frac{5}{2} + \frac{4}{1}\cdot\frac{2}{2}$$

$$(x-2)^2 = \frac{5}{2} + \frac{8}{2}$$

$$(x-2)^2 = \frac{13}{2}$$

$$x - 2 = \pm\sqrt{\frac{13}{2}}$$

$$x = 2 \pm \sqrt{\frac{13}{2}}$$

The Quadratic Formula:

We can solve $ax^2 + bx + c = 0$ for any a, b, c.

Assume $a \neq 0$ (Why? Cause then we don't have a quadratic)

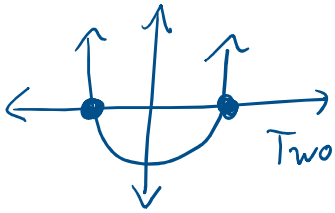
we can solve $ax^2+bx+c=0$

Assume $a \neq 0$ (Why? Cause then we don't have a quadratic)

$$ax^2+bx+c=0$$
$$-c \quad -c$$
$$ax^2+bx = -c$$
$$a\left(x^2+\frac{b}{a}x\right) = -c$$
$$x^2+\frac{b}{a}x = -\frac{c}{a}$$
$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2 = -\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$
$$\left(x+\frac{b}{2a}\right)^2 = -\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$
$$\left(x+\frac{b}{2a}\right)^2 = -\frac{c}{a}+\frac{b^2}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)^2 = -\frac{c\cdot 4a}{4a}+\frac{b^2}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$
$$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$
$$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow \text{Quadratic Formula}$$

- The discriminant of ax^2+bx+c is b^2-4ac. If the discriminant is
  - ↳ positive, then $ax^2+bx+c=0$ has 2 real solutions

    i.e.



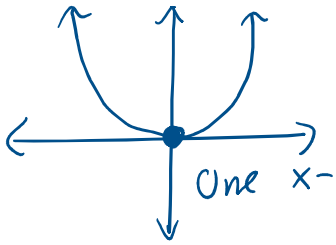
    Two intercepts b/c $b^2-4ac>0$

  - ↳ zero, then $ax^2+bx+c=0$ has 1 real solution

    i.e. ↑ ↑ ↑

↳ zero, then ax^2+bx+c has ...

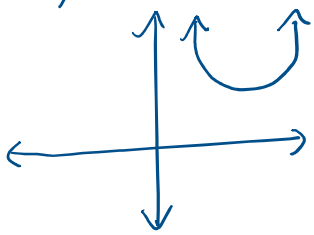
i.e.



One x-intercept b/c $b^2-4ac=0$

↳ negative, then $ax^2+bx+c<0$ has no real solutions

i.e.



No x-intercepts b/c $b^2-4ac<0$

Ex 2: For each equation, determine if there are one, two, or no solutions. If solutions exist, find them.

(a) $2x^2-5x-6=0$.

Idea: b^2-4ac and check if >0, $=0$ or <0

$b=-5$, $a=2$, $c=-6$

$(-5)^2+4(2)(+6)$

$=25+48=73>0 \Rightarrow$ Two real solutions

By quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-5)\pm\sqrt{73}}{2(2)}=\frac{5\pm\sqrt{73}}{4}$$

(b) $2x^2-5x+7=0$

Calculate b^2-4ac

$b=-5$, $a=2$, $c=7$

$=(-5)^2-4(2)(7)$

$=25-56<0 \Rightarrow$ No real solutions

$= (-5) - 4(2)(7)$

$= 25 - 56 < 0 \Rightarrow$ No real solutions

(c) $4x^2 = 36x - 81$

Rewrite $4x^2 - 36x + 81 = 0$

Calculate $b^2 - 4ac$

$b = -36,\ a = 4,\ c = 81$

$= (-36)^2 - 4(4)(81) = 0 \Rightarrow$ 1 real solution

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-36) \pm \sqrt{0}}{2(4)} = \frac{36}{8} = \frac{4 \cdot 9}{8} = \frac{9}{2}$$