Lesson 11: Graphs of Quadratic Functions

General Form, Graph Characteristics

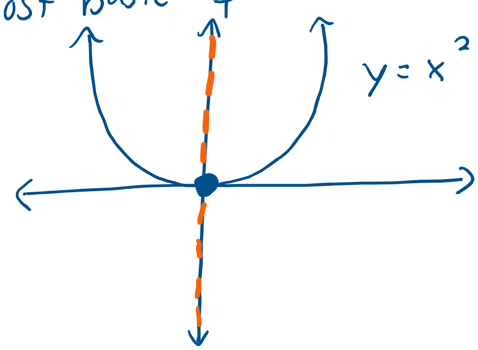
- A quadratic function is a function that can be written in the form

$$f(x) = ax^2 + bx + c$$

where a, b, c constants and $a \neq 0$

Note: For quadratic functions, we call $ax^2 + bx + c$ the general form

- Most basic quadratic function: $f(x) = x^2$



Characteristics:

- Vertex $(0,0)$
- Max/min: $y = 0$
  ↳ y-values/outputs
- Axis of symmetry: x-value of vertex
  $x = 0$
- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$

- Recall that we can rewrite a quadratic function by completing the square. Why? B/c it will help us graph the function

Ex 1: Let $f(x) = 2x^2 + 12x + 11$

ⓐ Rewrite $f(x)$ by completing the square.

$$f(x) = 2x^2 + 12x + \underline{\quad} + 11 - \underline{\quad}$$
$$= 2(x^2 + 6x + \underline{9} - \underline{9}) + 11$$

$\div 2 \rightarrow (\;)^2$

$$= 2(x^2 + 6x + 9) + 2(-9) + 11$$

$\div 2 \quad (\;)^2$

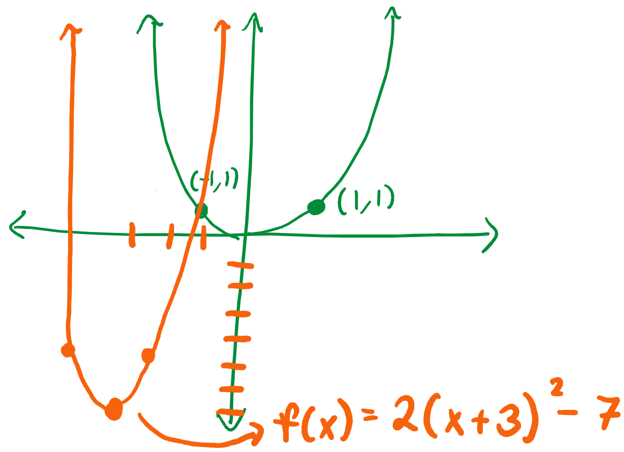
$= 2(x^2 + 6x + 9) + 2(-9) + 11$

$= 2(x+3)^2 - 18 + 11$

$= 2(x+3)^2 - 7$

ⓑ Graph $f(x)$ by using graph transformations. Identify the characteristic of the graph.

$f(x) = 2(x+3)^2 - 7$



$f(x) = x^2 \Rightarrow (x+3)^2$

↓ Shift to left by 3

$\Rightarrow 2(x+3)^2$

↓ Compression Vertically by 2

$\Rightarrow 2(x+3)^2 - 7$

↓ Shift down by 7.

Vertex: $(-3, -7)$

Max/Min: $y = -7$

Axis of Symmetry: $x = -3$

Domain: $(-\infty, \infty)$

Range: $[-7, \infty)$

Ex 2: Let $f(x) = -5x^2 + 10x - 7$

ⓐ Rewrite $f(x)$ by completing the square

$f(x) = -5(x^2 - 2x + \underline{1} - \underline{1}) - 7$

$\div 2 \quad (\;)^2$

$= -5(x^2 - 2x + 1) - 5(-1) - 7$

$= -5(x-1)^2 + 5 - 7$

$= -5(x-1)^2 - 2$

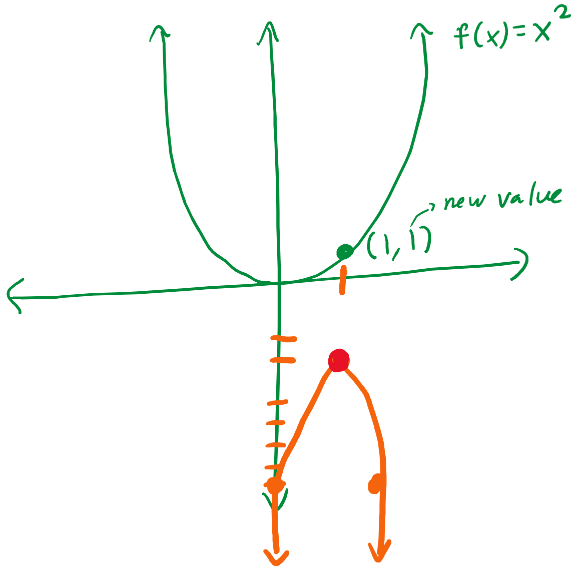
ⓑ Graph $y = f(x)$ and identify the following characteristic:

$f(x) = x^2 \Rightarrow (x-1)^2$

$f(x) = -5(x-1)^2 - 2$

(b) Graph $y = f(x)$ and identify the following

$f(x) = -5(x-1)^2 - 2$



$f(x) = x^2 \Rightarrow (x-1)^2$

↓ Shift to right by 1

$\Rightarrow -5(x-1)^2$

↓ Compressed vertically by 5

Reflection over x-axis

$\Rightarrow -5(x-1)^2 - 2$

↓ Shift down by 2.

Vertex: $(1, -2)$

Max/Min: $y = -2$

Axis of symmetry: $x = 1$

Domain: $(-\infty, \infty)$

Range: $(-\infty, -2]$

Standard / Vertex Form

We call $f(x) = a(x-h)^2 + k$ the standard form of a quadratic function (aka vertex form)

- Vertex is at (h, k)
- Axis of symmetry is $x = h$

Bonus fact: For general & vertex form

$a > 0 \Rightarrow$ graph opens up
$\Rightarrow$ function has min

$a < 0 \Rightarrow$ graph opens down
$\Rightarrow$ function has max

Form	General	Standard / Vertex
Equation	$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$
Vertex	$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	(h, k)
Axis of Symmetry	$x = -\frac{b}{2a}$	$x = h$
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$

Domain	$(-\infty, \infty)$		$(-\infty, \infty)$	
	$a>0$	$a<0$	$a>0$	$a<0$
Min	$f(-\frac{b}{2a})$	None	k	None
Max	None	$f(-b/2a)$	None	k
Range	$[f(-\frac{b}{2a}), \infty)$	$(-\infty, f(\frac{-b}{2a})]$	$[k, \infty)$	$(-\infty, k]$

Intercepts & Zeros

- A point $(0, v)$ on the graph of f is called a y-intercept of f
  - ↳ Plug in $x=0$.
- A point $(u, 0)$ on the graph of f is called a x-intercept of f.
  - ↳ Solve for $y=0$.
  - ↳ Another way to say this is find zeros or roots of f.

Ex 3: Let $f(x) = 3x^2 - 6x - 24$

ⓐ Find the vertex.

$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ [by Table]

$\left(\frac{-(-6)}{2(3)}, f\left(\frac{-(-6)}{2(3)}\right)\right)$

$(1, f(1))$ $\quad f(1) = 3 - 6 - 24 = -27$

$(1, -27)$

ⓑ Does the graph have a max/min? What is it?

Is $a > 0$ or $a < 0$?

⇓

Min ⇒ Equals the y-coordinate of vertex

$y = -27$

ⓒ What is y-intercept?

Let $x = 0 \Rightarrow y = -24$

$(0, -24)$

ⓓ What is x-intercept?

Let $y = 0$.

$3x^2 - 6x - 24 = 0$

$3(x^2 - 2x - 8) = 0$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

$(4, 0), (-2, 0)$

ⓔ What is zeroes?

$x = 4, -2$

Ex 4: Find an eqn for the parabola with vertex

[illegible] and passing through the point $(-2, 14)$.

Ex 7. Find an eqn for the parabola with vertex
(1, -4) and passing through the point (-2, 14).

Vertex Form: $f(x) = a(x-h)^2 + k$

$f(x) = a(x-1)^2 - 4$ b/c (1, -4) is vertex

Now find a w/ (-2, 14).

$$14 = a(-2-1)^2 - 4$$
$$+4 \qquad\qquad +4$$
$$18 = a(-3)^2$$
$$\frac{18}{9} = \frac{a \cdot 9}{9}$$
$$a = 2 \implies f(x) = 2(x-1)^2 - 4$$