Wednesday, February 18, 2026 11:45 AM

Lesson 14: Higher-order polynomials

- A few more features to analyze polynomials
  - ↳ End Behavior
  - ↳ Multiplicity of zeros

Ex 1: Identify the following for each polynomial

- leading term
- degree
- constant term

ⓐ $g(x) = x^5(x-2)(3x^2+1)(4x-5)(2-x)$

$= x^5(x-2)(3x^2+1)(4x-5)(-x+2)$

Leading term: $x^5 \cdot x \cdot 3x^2 \cdot 4x \cdot (-x) = -12x^{10}$

Degree (attached to leading term): 10

Constant term: $0(-2)(1)(-5)(2) = 0$

$g(x) = (x^5 - 0)(x-2)(3x^2+1)(4x-5)(-x+2)$

ⓑ $p(x) = (x-4)(3x+2)(2x^3+1)$

Leading term: $6x^5$

Degree: 5

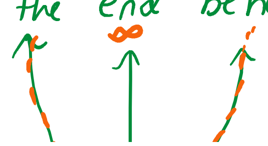
Constant: $(-4)(2)(1) = -8$

End Behavior

The end behavior of a function describes what happens to $f(x)$ when x approaches ∞ or $-\infty$.

Ex 2: Describe the end behavior of the following functions:

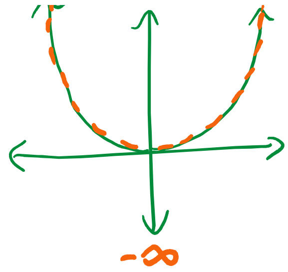
ⓐ $f(x) = x^2$



As x approaches ∞ $(x \to \infty)$, Right

$f(x) \to \infty$

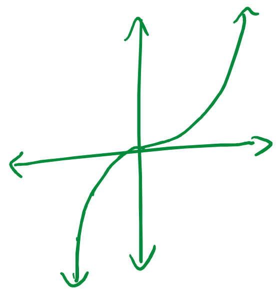
ⓐ $f(x) = x^2$



As x approaches ∞ $(x \to \infty)$,
$f(x) \to \infty$

As x approaches $-\infty$ $(x \to -\infty)$ ↗ Left
$f(x) \to \infty$

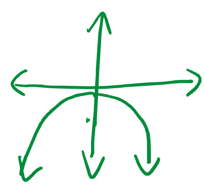
ⓑ $f(x) = x^3$



As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

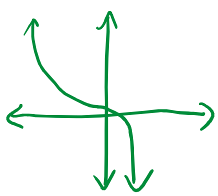
ⓒ $f(x) = -2x^2$



As $x \to -\infty$, $f(x) \to -\infty$

As $x \to +\infty$, $f(x) \to -\infty$

ⓓ $f(x) = -4x^3$



As $x \to -\infty$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to -\infty$

In general for $f(x) = kx^n$

	Even Power	Odd power
Positive constant $k>0$	$x \to \infty$, $f(x) \to \infty$ $x \to -\infty$, $f(x) \to \infty$ (Look @ (a))	$x \to -\infty$, $f(x) \to -\infty$ $x \to \infty$, $f(x) \to \infty$ (Look @ (b))
Negative constant $k<0$	$x \to \infty$, $f(x) \to -\infty$ $x \to -\infty$, $f(x) \to -\infty$ (Look @ (c))	$x \to -\infty$, $f(x) \to \infty$ $x \to \infty$, $f(x) \to -\infty$ (Look @ (d))

The end behavior of a polynomial depends only on the leading term

Ex 3: Describe the end behavior of the following:

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ⓐ $y = 5 - 4x^6$

Leading term is $-4x^6 \Rightarrow$ degree is $6 \Rightarrow 6$ is even

But -4 for k

$x \to \infty,\ f(x) \to -\infty$

$x \to -\infty,\ f(x) \to -\infty$

Multiplicity

- When a polynomial is fully factored, the # of times a particular factor appears is called its multiplicity.

$f(x) = (x-1)^3 = (x-1)(x-1)(x-1) \Rightarrow$ multiplicity is 3

Ex 4: Find the multiplicity of each linear factor of $P(x)$:

$P(x) = 2(x+3)^2(x-4)^3(x+1)^7\underbrace{(x-2)^2}(x-5)\underbrace{(x-2)}$

combine

$= 2(x+3)^2(x-4)^3(x+1)^7(x-2)^3(x-5)^1$

$(x+3)^2$	$(x-4)^3$	$(x+1)^7$	$(x-2)^3$	$(x-5)^1$
$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$x=-3$	$x=4$	$x=-1$	$x=2$	$x=5$
Multiplicity of 2	Multi. of 3	Multi. of 7	Multi. of 3	Multi. of 1

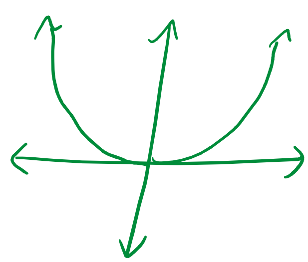
The multiplicity of a zero k of $f(x)$ is related to how the graph looks near $(k, 0)$.

Ex 5: ⓐ $f(x) = x^2 = (x-0)^2$

$\Downarrow$

$x = 0$

multiplicity of 2



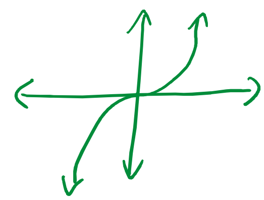
$x = 0$
multiplicity of 2

When multiplicity is <u>even</u> it <u>touches</u> the x-axis

ⓑ $f(x) = x^3 = (x-0)^2$

$\Downarrow$

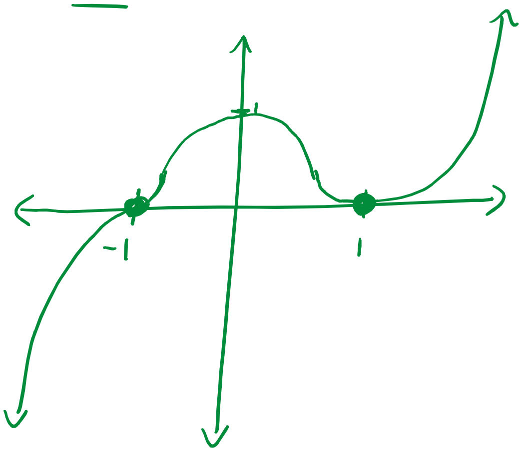
$x = 0$
multiplicity of 3



When multiplicity is <u>odd</u> it crosses the x-axis.

ⓒ $f(x) = (x+1)(x-1)^2 = 0$

$(x+1)$	$(x-1)^2$
$\Downarrow$	$\Downarrow$
$x = -1$	$x = 1$
Multi. of 1	Multi. of 2
$\Downarrow$	$\Downarrow$
Crosses @ $x = -1$	Touches @ $x = 1$



Leading term: $x \cdot x^2 = x^3$

$\downarrow$

End Behavior $x \to -\infty,\ f(x) \to -\infty$
$x \to +\infty,\ f(x) \to \infty$

y-intercept $1 \cdot (-1)^2 = 1$