Wednesday, March 4, 2026 11:30 AM

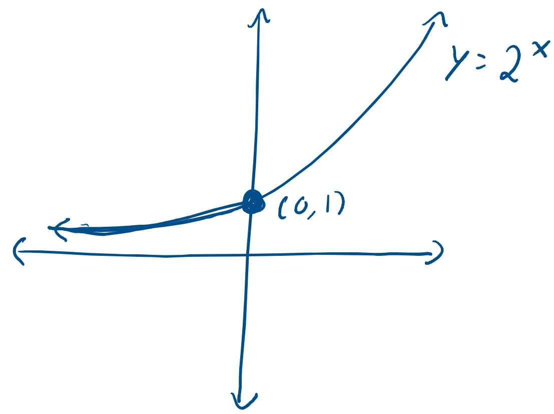
Lesson 19: Exponential Functions

Exponential Function

Ex. $f(x) = 2^x$

x	$y = 2^x$
-3	$1/8$
-2	$1/4$
-1	$2^{-1} = 1/2$
0	1
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

$\times 2$ $\div 2$



Domain: $(-\infty, \infty)$

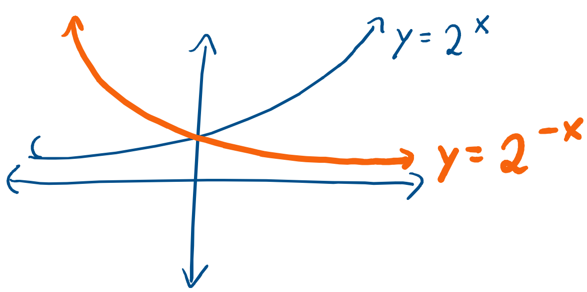
Range: $(0, \infty)$

HA: $y = 0$

Horizontal Line Test: Passes $\Rightarrow$ Inverse

End Behavior: $x \to -\infty$ $f(x) \to 0$
$x \to \infty$ $f(x) \to \infty$

Ex: $g(x) = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$ Horizontal Reflection



Exponential function are $f(x) = a \cdot b^x$ where $a \neq 0$, $b > 0$, $b \neq 1$

- ↳ If $b > 1$, then its exponential growth
- ↳ If $0 < b < 1$, then its exponential decay
- ↳ $f(0) = a$ (initial value)

Example 1: Is the base appropriate for an exponential function?

(a) $f(x) = 2^x$

Check that $f(x) = a \cdot b^x$ where $a \neq 0$, $b > 0$, $b \neq 1$

$f(x) = 2^x = 1 \cdot 2^x$

So $a = 1$ ✓ $b = 2$ ✓

(b) $f(x) = (\sqrt{2})^x$

Check that $f(x) = a \cdot b^x$ where $a \neq 0$, $b > 0$, $b \neq 1$

$f(x) = 1 \cdot (\sqrt{2})^x$

So $a = 1$ ✓ $b = \sqrt{2}$ ✓

(c) $f(x) = (-4)^x$

Check that $f(x) = a \cdot b^x$ where $a \neq 0$, $b > 0$, $b \neq 1$ } **Not exponential**

$a = 1$ ✓ $b = -4$ ✗

(d) $f(x) = -1(4)^x$

Check that $f(x) = a \cdot b^x$ where $a \neq 0$, $b > 0$, $b \neq 1$

$a = -1$ ✓ $b = 4$ ✓

Recap different functions

↳ Exponential $f(x) = ab^x$
independent variable is in the power

↳ Power functions $f(x) = k \cdot x^p$
independent variable is in the base

↳ Linear functions $(y = mx + b)$
The change in y is the same amount over the change of x.

The change in y is the same amount over the change of x.

Ex 2: Does the statement represent an exponential function.

ⓐ A population doubles in size every 10 years.

Pop is 10 @ 10 years
20 @ 20 years
40 @ 40 years

$\} \; 10 \cdot 2^x \to$ Exponential

ⓑ A child purchase 4 new video games every year.

Year 1 4
Year 2 8
Year 3 12

$\} \; x+4 \Rightarrow$ Linear

ⓒ A radioactive element $\underbrace{\text{decay}}$ by 0.25% per year

$\Downarrow$
Exponential

Ex 3: Simplify

ⓐ $e^{3x}e^{-5x} = e^{3x+(-5x)} = e^{-2x}$

ⓑ $(e^{2x})^6 = e^{2x\cdot 6} = e^{12x}$

ⓒ $\dfrac{e^{-10}}{e^{-3}} = e^{-10-(-3)} = e^{-7}$

ⓓ $(e^{5x}+e^{-2x})(e^{5x}-e^{-2x}) = e^{10x} + \cancel{e^{3x}} - \cancel{e^{3x}} - e^{-4x}$

$= e^{10x} - e^{-4x}$

Way 1:

	e^{5x}	$-e^{-2x}$
e^{5x}	$e^{5x}e^{5x}$	$-e^{5x}e^{-2x}$
e^{-2x}	$e^{5x}e^{-2x}$	$-e^{-2x}e^{-2x}$

Way 2: Note the paranthesis are the same but one has a subtraction

$(a-b)(a+b) = a^2 - b^2$

$= (e^{5x})^2 - (e^{-2x})^2$

$$(a-b)(a+b) = a^{\frown} - b^{\frown}$$
$$= (e^{5x})^2 - (e^{-2x})^2$$
$$= e^{10x} - e^{-4x}$$

Compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A - dollars
t - years

Initial Investment : P (principal)

Interest rate (decimal) : r

times compounded per year : n

Ex 4: Suppose you invest \$10,000 in an account that earns 5% interest.

ⓐ How much money will you have in the account in 8 years if the interest is compounded

ⓘ monthly? $(n=12)$

$P = 10{,}000 \qquad r = 0.05$

$$A = 10{,}000\left(1 + \frac{0.05}{n}\right)^{n \cdot t}$$

$$A = 10{,}000\left(1 + \frac{0.5}{12}\right)^{12 \cdot t}$$

$$A(8) = 10{,}000\left(1 + \frac{0.5}{12}\right)^{12 \cdot 8} \approx 14905.85$$

ⓑ weekly? $(n=52)$

$$A = 10{,}000\left(1 + \frac{0.05}{n}\right)^{n \cdot t}$$

$$= 10{,}000\left(1 + \frac{0.05}{52}\right)^{52 \cdot t}$$

$$A(8) = 10{,}000\left(1 + \frac{0.5}{12}\right)^{52 \cdot 8} \approx 14915.38$$

(b) Which compounding period resulted in more money?

Weeks

If interest is compounded continuously, then $A = Pe^{rt}$

Ex 5: Suppose you invest \$10,000 in an account that earns 5% interest. How much money will you have in the account in 8 years if the interest is compounded continuously?

$P = 10,000$ $\quad r = 0.05$ $\quad$ continuous $\Rightarrow A = Pe^{rt}$

$A = 10,000\, e^{0.05t}$

$A(8) = 10,000\, e^{0.05(8)} \approx 14918.25$

Ex 6: A sample of a radioactive element decays according to the model $N(t) = 40e^{-0.2t}$ where $N(t)$ is the amount remaining in grams and t is in days.

(a) What is the initial mass of the sample?

i.e. What is $N(0)$?

$N(0) = 40e^{-0.2(0)} = 40$

(b) What mass will remain after 36 hours? Round to 2 decimal places.

36 hrs $\Rightarrow$ 1.5 days (Remember t is in days)

$N(1.5) = 40\, e^{-0.2(1.5)} \approx 29.63$ g