Monday, March 9, 2026 11:33 AM

Lesson 21: Properties of Logarithms

Final Exam
5/5
@ 8am-10am

- Log rules $(b>0,\ b\neq 1,\ x>0,\ y>0)$ $\quad y=\log_b(x)$

① Product Rule

$$\log_b(xy)=\log_b(x)+\log_b(y)$$

② Quotient Rule

$$\log_b\left(\frac{x}{y}\right)=\log_b(x)-\log_b(y)$$

③ Power Rule

$$\log_b(x^y)=y\log_b(x)$$

- Special Inputs

① Input = 1 $\quad y=\log_b(1)=0$

$$y=\ln(1)=0$$

B/c log and exponentials are inverses

② Input = Base $\quad y=\log_b(b)=1$

$$y=\ln(e)=1$$

- Inverse properties

① $\log_b(b^x)=x$ $\qquad$ ② $b^{\log_b(x)}=x$

$\ln(e^x)=x$ $\qquad$ $e^{\ln(x)}=x$

Some algebra recall

- $\dfrac{1}{x}=x^{-1}$
- $\dfrac{1}{x^m}=x^{-m}$
- $\sqrt{x}=x^{1/2}$
- $\sqrt[q]{x^p}=x^{p/q}$

Expanding Logarithmic Expressions

Ex 1: Express the following as sums, differences, and multiples of logs. (Do not leave any negative exponents) logs. (Do not leave any negative exponents)

ⓐ $\log(x^2y) = \log(x^2) + \log(y)$

$= 2\log(x) + \log(y)$

ⓑ $\log_2\left(\frac{w^2}{\sqrt{z}}\right) = \log_2(w^2) - \log_2(\sqrt{z})$

$= \log_2(w^2) - \log_2(z^{1/2})$

$= 2\log_2(w) - \frac{1}{2}\log_2(z)$

ⓒ $\ln\left(\frac{x^{16}z^4}{y^4w^7}\right) = \ln(x^{16}z^4) - \ln(y^4w^7)$

$= \left[\ln(x^{16}) + \ln(z^4)\right] - \left[\ln(y^4) + \ln(w^7)\right]$

$= \ln(x^{16}) + \ln(z^4) - \ln(y^4) - \ln(w^7)$

$= 16\ln(x) + 4\ln(z) - 4\ln(y) - 7\ln(w)$

ⓓ $\ln\left(\frac{e^4}{\sqrt[5]{x^3}}\right) = \ln(e^4) - \ln(\sqrt[5]{x^3})$

$= \cancel{\ln(e}^4) - \ln(x^{3/5})$

$= 4 - \ln(x^{3/5})$

$= 4 - \frac{3}{5}\ln(x)$

ⓔ $\log_3\left(\frac{\sqrt{x}}{27y}\right) = \log_3(\sqrt{x}) - \log_3(27y)$

$= \log_3(\sqrt{x}) - \left[\log_3(27) + \log_3(y)\right]$

$= \log_3(\sqrt{x}) - \log_3(27) - \log_3(y)$

$= \log_3(x^{1/2}) - \log_3(3^3) - \log_3(y)$

$$= \log_3(x^{1/2}) - \cancel{\log_3(3}^{3}) - \log_3(y)$$

$$= \log_3(x^{1/2}) - 3 - \log_3(y)$$

$$= \frac{1}{2}\log_3(x) - 3 - \log_3(y)$$

(f) $\ln\left(\frac{x+5}{x-6}\right) = \ln(x+5) - \ln(x-6)$

Stop Here!

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(xy) = \ln(x) + \ln(y) \neq \ln(x+y)$$

Condensing Logarithmic Expressions

Ex 2: Condense into a single logarithm with a leading coefficient of 1.

(a) $2\log_3(x) - 2\log_3(y) + 3\log_3(z)$

$$= \log_3(x^2) - \log_3(y^2) + \log_3(z^3)$$

$$= \log_3\left(\frac{x^2 z^3}{y^2}\right)$$

$$(x^a)^b = x^{a \cdot b}$$

(b) $2\ln\left(\frac{x}{y}\right) + \frac{1}{2}\ln(x^4 y^{12}) - 3\ln(xy)$

$$= \ln\left(\left(\frac{x}{y}\right)^2\right) + \ln\left((x^4y^{12})^{1/2}\right) - \ln\left((xy)^3\right)$$

$$= \ln\left(\frac{x^2}{y^2}\right) + \ln(x^2y^6) - \ln(x^3y^3)$$

$$= \ln\left(\frac{x^2 \cdot x^2 y^6}{\dots}\right)$$

$$= \ln\left(\frac{x^2 \cdot x^2 y^6}{y^2 \cdot x^3 y^3}\right)$$

$$= \ln\left(\frac{x^4 y^6}{x^3 y^5}\right)$$

$$= \ln\left(x^{4-3} y^{6-5}\right)$$

$$= \ln(xy)$$

(c) $13 + 3\ln(x) - 2\ln(2x-3)$

$13 = \ln(e^{13})$

Idea is $= \ln(\quad)$

$$= \ln(e^{13}) + 3\ln(x) - 2\ln(2x-3)$$

$$= \ln(e^{13}) + \ln(x^3) - \ln((2x-3)^2)$$

$$= \ln\left(\frac{e^{13}x^3}{(2x-3)^2}\right)$$

(d) $9\ln(x-8) + \frac{1}{9}\ln(x^{54}) - 18\ln((x+12)^{10})$

$$= \ln((x-8)^9) + \ln((x^{54})^{1/9}) - \ln(((x+12)^{10})^{18})$$

$$= \ln((x-8)^9) + \ln(x^6) - \ln((x+12)^{180})$$

$$= \ln\left(\frac{(x-8)^9 x^6}{(x+12)^{180}}\right)$$