# Lesson 22: Exponential and Logarithmic Equations

## General Strategies

- Use $b^\wedge$ to undo $\log_b$
- Use $\log_b$ to undo $b^\wedge$

} Why? B/c they are inverses

- $\log_b(x) = \log_b(y) \Rightarrow x = y$
- $b^x = b^y \Rightarrow x = y$

} B/c one-to-one and again inverses

- This is true when $b$ is $e$ and $\log_b$ is ln.
- Sometimes you will find extraneous solutions, so always check your domain from original statement.

## Solving Logarithmic Equations

Ex 1: Solve for $x$, if possible.

ⓐ $\log_2(x+3) = 4$

$\cancel{2} \qquad 2$

$x+3 = 2^4$

$x+3 = 16$

$x = 13$ ✓

Check if $x=13$ works in the original statement.

$\log_2(x+3) \stackrel{?}{=} 4$

$\log_2(13+3) = \log_2(16)$

$= \log_2(2^4)$

$= 4$ ✓

ⓑ $\ln(3x) = -1$

$e \qquad e$

$3x = e^{-1}$

Check $x = \frac{1}{3e}$ works in the original Statement

$\ln(3x) \stackrel{?}{=} -1$

$$3x = e^{-1}$$
$$x = \frac{e^{-1}}{3}$$
$$x = \frac{1}{3e} \checkmark$$

$$\ln(3x) \stackrel{?}{=} -1$$
$$\ln\left(3\left(\frac{1}{3e}\right)\right) = \ln\left(\frac{1}{e}\right)$$
$$= \ln(e^{-1})$$
$$= -1 \checkmark$$

ⓒ $\log(x+9) + \log(x) = 1$

$$\log((x+9)(x)) = 1$$

$$(x+9)(x) = 10^1$$
$$x^2 + 9x = 10$$
$$x^2 + 9x - 10 = 0$$
$$(x+10)(x-1) = 0$$
$$x = \cancel{-10}, 1$$

Answer: $x = 1$

Check $x=-10, 1$ works in the original statement [logs or lns can't have negatives inside]

$x=-10$: $\log(x+9) + \log(x) = 1$

$$\log(-1) + \log(-10)$$

$x = -10$ is an extraneous solution

why b/c no negatives in log.

$x=1$: $\log(x+9) + \log(x) = 1$

$$\log(1+9) + \log(1)$$
$$= \log(10^1) + \cancel{\log(1)}^{0}$$
$$= 1 \checkmark$$

ⓓ $\log_3(x-4) + \log_3(x+15) = \log_3(x^2)$

$$\cancel{\log_3}((x-4)(x+15)) = \cancel{\log_3}(x^2)$$

$$(x-4)(x+15) = x^2$$
$$x^2 - 4x + 15x - 60 = x^2$$
$$x^2 + 11x - 60 = x^2$$
$$-x^2 \qquad\qquad -x^2$$

Check $x = \frac{60}{11}$ works.

$$\log_3\underbrace{(x-4)}_{>0} + \log_3\underbrace{(x+15)}_{>0}$$
$$= \log_3\underbrace{(x^2)}_{>0}$$

$$x^2 + 11x - 60 = x^2$$
$$-x^2 \qquad -x^2$$
$$11x - 60 = 0$$
$$x = \frac{60}{11} = 5\tfrac{5}{11} \checkmark$$

ⓔ $\log_4(x^2+9) - \log_4(x+1) + 5 = 6$
$$-5 \quad -5$$

$$\log_4(x^2+9) - \log_4(x+1) = 1$$
$$\log_4\left(\frac{x^2+9}{x+1}\right) = 1$$
$$4 \qquad 4$$
$$\frac{x^2+9}{x+1} = 4^1$$
$$x^2+9 = 4(x+1)$$
$$x^2+9 = 4x+4$$
$$x^2-4x+5=0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$\sqrt{16-20}$

$\sqrt{-4}$

$\Downarrow$

No real solutions.

## Solving Exponential Equations

Ex 2: Solve for x, if possible

ⓐ $3^{x+1} = 27$
$$3^{x+1} = 3^3$$
$$\log_3(3^{x+1}) = \log_3(3^3)$$

$$\log_3(3^{x+1}) = \log_3(3^3)$$

$$x+1 = 3$$

$$x = 2$$

ⓑ $24 \cdot 2^{3-x} = 3$

$$2^{3-x} = \frac{3}{24}$$

$$2^{3-x} = \frac{1}{8}$$

$$2^{3-x} = 8^{-1}$$

$$2^{3-x} = (2^3)^{-1}$$

$$2^{3-x} = 2^{-3}$$

$$3 - \cancel{x} = -\cancel{3}$$

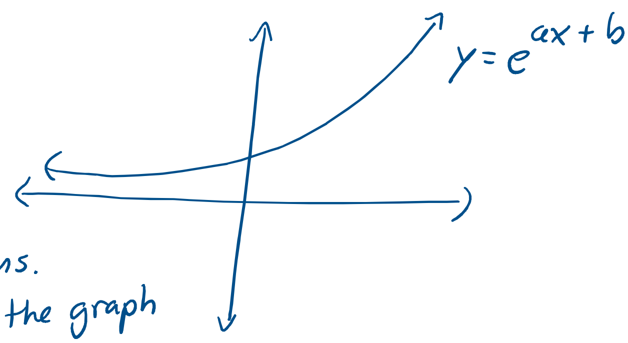
$$+3 \ \cancel{+x} \qquad \cancel{+3} \ +x$$

$$6 = x$$

ⓒ $e^{x+2} + 7 = 1$

$$-7 \quad -7$$

$$e^{x+2} = -6$$ This never happens. b/c the graph




$$\ln(e^{x+2}) = \ln(-6)$$

$$x+2 = \underbrace{\ln(-6)}$$

This implies no solution.

ⓓ $10^{5-2x} + 1 = 6$

$$-1 \quad -1$$

$$10^{5-2x} = 5$$

$$10^{5-2x} = 5$$

$$\log(10^{5-2x}) = \log(5)$$

$$5-2x = \log(5)$$

$$-\log(5) \quad +2x \quad -\log(5) \quad +2x$$

$$\frac{-\log(5)+5}{2} = \frac{2x}{2}$$

$$\frac{-\log(5)+5}{2} = x$$

(e) $2^{3x+9} = 5^{8x+9}$

$$\ln(2^{3x+9}) = \ln(5^{8x+9})$$

$$(3x+9)\ln(2) = (8x+9)\ln(5)$$

$$3\ln(2)x + \cancel{9\ln(2)} = \cancel{8\ln(5)x} + 9\ln(5)$$

$$-8\ln(5)x \quad \cancel{-9\ln(2)} \quad \cancel{-8\ln(5)x} \quad -9\ln(2)$$

$$3\ln(2)x - 8\ln(5)x = 9\ln(5) - 9\ln(2)$$

$$[3\ln(2) - 8\ln(5)]x = 9\ln(5) - 9\ln(2)$$

$$x = \frac{9\ln(5) - 9\ln(2)}{3\ln(2) - 8\ln(5)}$$

(f) $e^{2x} - 3e^{x} - 54 = 0$

$$(e^{x})^2 - 3e^{x} - 54 = 0$$

$54$

$\wedge$

$-9+6$

Let $u = e^{x}$ $\longrightarrow$ $u = e^{x}$

$$u^2 - 3u - 54 = 0$$

$$u^2 - 9u + 6u - 54 = 0$$

$$u(u-9) + 6(u-9) = 0$$

$u = -6$

$e^{x} = -6$

~~$x = \ln(-6)$~~

$u = 9$

$e^{x} = 9$

$x = \ln(9)$ ✓

$u$

$u(u-9)+6(u-9)=0$

$(u+6)(u-9)=0$

$u=-6,9$

$\cancel{x=\ln(-6)}$

No!

$x=\ln(4)$ ✓