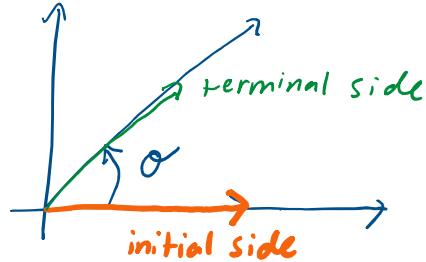


Written HW due this week via email

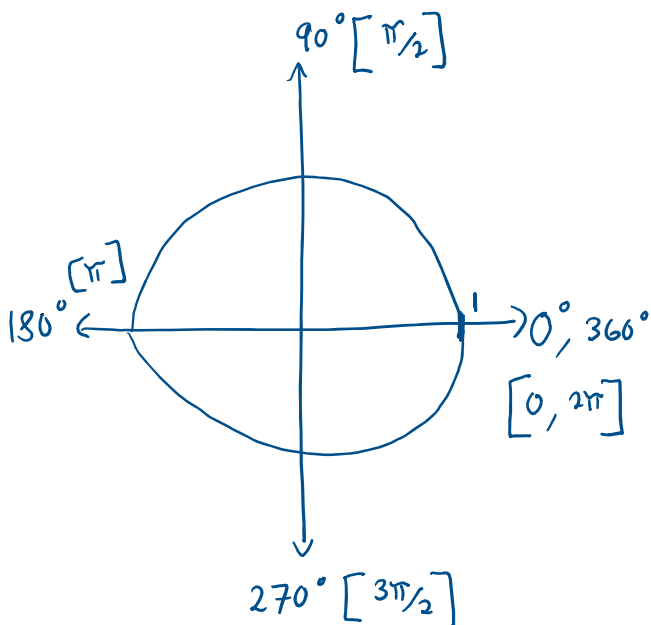
Lesson 23' Angles



When discussing angles, amount rotated is counter clockwise around a circle, starting at the positive x-axis

Degrees: There are 360 degrees (360°) in one revolution around a circle

Radians: Recall that the circumference of a circle with radius r is $2\pi r$. So radian is the corresponding multiple of the radius we have traveled around the circle.



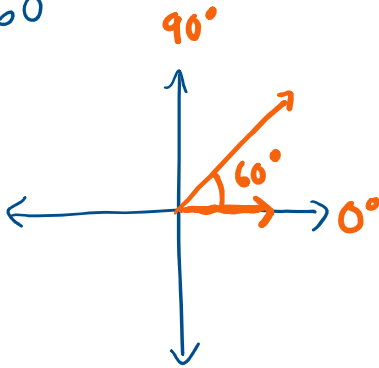
Proportion of Circle	Angle in degrees	Angle in radians
$1/12$	$\frac{360}{12} = 30^\circ$	$\frac{2\pi}{12} = \frac{\pi}{6}$
$1/8$	45°	$\pi/4$
$1/6$	60°	$\pi/3$
$1/4$	90°	$\pi/2$
$3/8$	135°	$3\pi/4$
$1/2$	180°	π
$5/8$	225°	$5\pi/4$

↓
 $270^\circ [3\pi/2]$

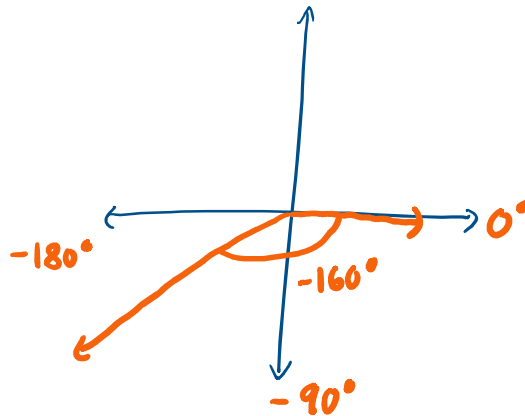
$5/8$	225°	$5\pi/4$
$3/4$	270°	$3\pi/2$
$7/8$	315°	$7\pi/4$
1	360°	2π

Ex 1: Draw an angle in standard position with the given measure,

(a) 60°

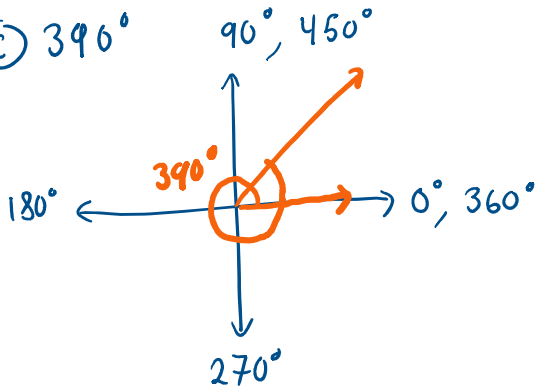


(b) -160°



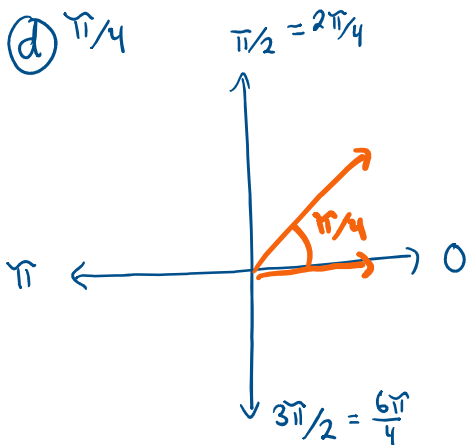
Note: When there is a negative just go the opposite way

(c) 390°

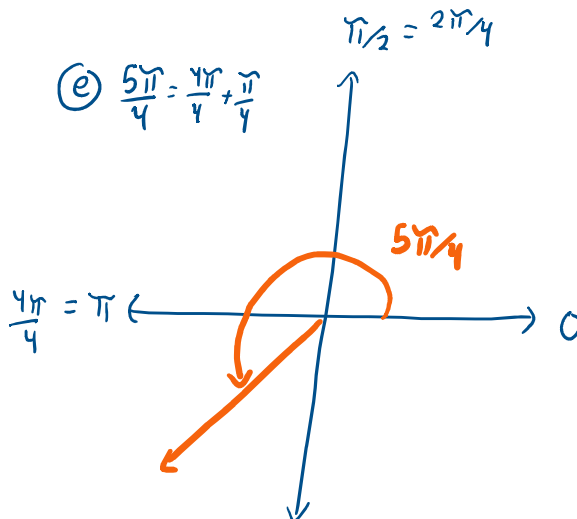


$390^\circ = 360^\circ + 30^\circ \rightarrow$ Leftover
 ↓
 1 full revolution

(d) $\pi/4$

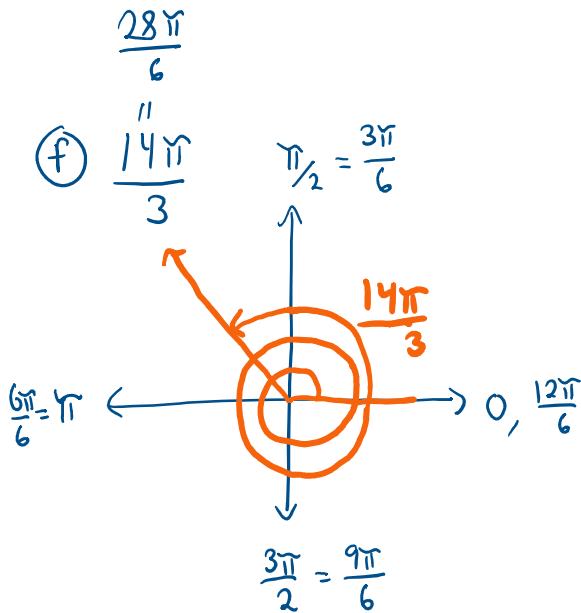


(e) $\frac{5\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4}$



$$\downarrow 3\pi/2 = \frac{6\pi}{4}$$

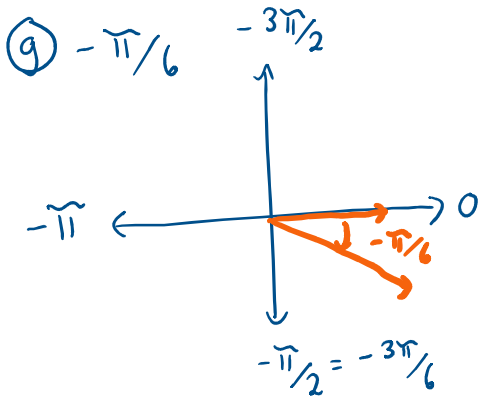
$$\downarrow 3\pi/2 = 6\pi/4$$



One revolution is $2\pi = \frac{12\pi}{6}$

$$\frac{28\pi}{6} = \frac{12\pi}{6} + \frac{16\pi}{6} = \frac{12\pi}{6} + \frac{12\pi}{6} + \frac{4\pi}{6}$$

↑ 1 revolution
 ↑ 2 revolutions



Great exam question is given (4 angles) which are in quadrant _____?

Conversion between radians & degrees

$$\frac{180^\circ}{\pi \text{ rad}} = 1$$

$$\frac{\pi \text{ rad}}{180^\circ} = 1$$

Ex 2: Convert the angles from degrees to radians

(a) $\theta = 70^\circ$

$$70^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{70\pi}{180} \text{ rad}$$

7...

Note: B/c I want radians so degrees need to cancel

$$\frac{180}{180} = \frac{180}{180}$$

$$= \frac{7\pi}{18}$$

degrees need to
cancel

(b) $\theta = -150^\circ$

$$-150^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{-150\pi}{180} \text{ rad}$$

$$= \frac{-15\pi}{18} = -\frac{5\pi}{6}$$

Ex 3: Convert the angles from radians to degrees.

(a) $\theta = \frac{2\pi}{3}$

$$\frac{2\pi}{3} \cdot \frac{180^\circ}{\pi} = \frac{2 \cdot 180^\circ}{3} = 2 \cdot 60^\circ = 120^\circ$$

(b) $\theta = \frac{5\pi}{7}$

$$\frac{5\pi}{7} \cdot \frac{180^\circ}{\pi} = \frac{5 \cdot 180^\circ}{7} = \frac{900^\circ}{7}$$

Two angles have the same initial and terminal sides are coterminal angles.

Ex 4: Find the angle between 0° and 360° that is coterminal to the given angle.

Tip: Add/subtract by 360° or 2π until you are between 0° and 360° .

(a) $\theta = -55^\circ$

$$-55^\circ + 360^\circ = 305^\circ \checkmark$$

(c) $\theta = 1020^\circ$

$$1020^\circ - 360^\circ = 660^\circ$$

$$660^\circ - 360^\circ = (300^\circ) \checkmark$$

$$-55^\circ + 360^\circ = 305^\circ \checkmark$$

$$1020 - 360 = 660$$
$$660^\circ - 360^\circ = \textcircled{300^\circ} \checkmark$$

$$\textcircled{b} \theta = 575^\circ$$

$$575^\circ - 360^\circ = 215^\circ \checkmark$$

Ex 5: Find the angle between 0 and 2π radians that is coterminal to the given angle.

$$\textcircled{a} \theta = -\frac{\pi}{4}$$

$$-\frac{\pi}{4} + \frac{2\pi}{1} = -\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}$$

$$\textcircled{b} \theta = \frac{11\pi}{3} \text{ [Checking that you are b/w } 0, 2\pi \Leftrightarrow 0, \frac{6\pi}{3}\text{]}$$

$$\frac{11\pi}{3} - 2\pi = \frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3} \checkmark$$

$$\textcircled{c} \theta = \frac{58\pi}{9} \text{ [Checking that you are b/w } 0, 2\pi \Leftrightarrow 0, \frac{18\pi}{9}\text{]}$$

$$\frac{58\pi}{9} - \frac{18\pi}{9} = \frac{40\pi}{9}$$

$$\frac{40\pi}{9} - \frac{18\pi}{9} = \frac{22\pi}{9}$$

$$\frac{22\pi}{9} - \frac{18\pi}{9} = \textcircled{\frac{4\pi}{9}} \checkmark$$