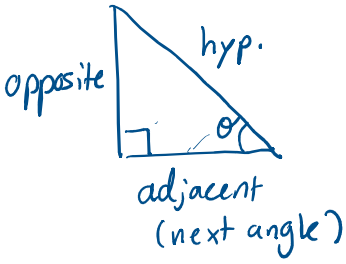
Lesson 24: Trigonometric Functions

Trig Ratios for acute angles

- A right triangle is a triangle that has 90° angle
  - ↳ 3 sides has special names (hypotenuse, adjacent, opposite)
    - hypotenuse: across 90° angle
    - adjacent, opposite: depended on θ



length (opp) ≤ length (hyp)
length (adj) ≤ length (hyp)

Sine [sin] $\sin\theta = \frac{opp}{hyp}$

Cosine [cos] $\cos\theta = \frac{adj}{hyp}$

tangent [tan] $\tan\theta = \frac{opp}{adj}$

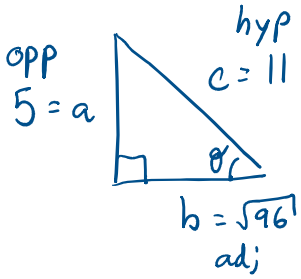
Reciprocal trig ratios

cosecant [csc] $\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp}$

secant [sec] $\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$

cotangent [cot] $\cot\theta = \frac{adj}{opp}$

Ex 1: For the following triangle, find the value of all six trig functions for the acute angle θ if $a = 5$ and $c = 11$



Pythagorean thm: $a^2 + b^2 = c^2$

$5^2 + b^2 = 11^2$

$25 + b^2 = 121$

$b^2 = 96$

$b = \sqrt{96}$

$\sin\theta = \frac{opp}{hyp} = \frac{5}{11}$ $\tan\theta = \frac{opp}{adj} = \frac{5}{\sqrt{96}}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{11}{5}$

$\cos\theta = \frac{adj}{\quad} = \frac{\sqrt{96}}{\quad}$ $\cot\theta = \frac{adj}{opp} = \frac{\sqrt{96}}{5}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{11}{\sqrt{96}}$

$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{96}}{11}$ $\quad \cot\theta = \frac{adj}{opp} = \frac{\sqrt{96}}{5}$ $\quad \sec\theta = \frac{1}{\cos\theta} = \frac{11}{\sqrt{96}}$

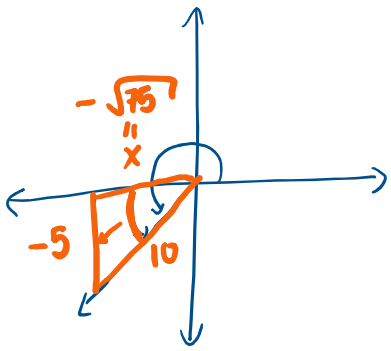
Trig Ratios for other Angles

Can find sin, cos, etc for $0° \le \theta < 90°$. What about other angles?

Reference angles come to rescue.

Ex 2: Find the exact value of the remaining six trig functions for an acute angle θ in the given quadrant.

$\sin(\theta) = -\frac{5}{10}$ with θ in Quadrant III



$\sin\theta = \frac{-5}{10} = -\frac{5}{10} = \frac{opp}{hyp}$

Find x w/ the pythagorean thm

$(-5)^2 + x^2 = 10^2$

$25 + x^2 = 100$

$x^2 = 75$

$x = \pm\sqrt{75}$

→ neg b/c x is on the left

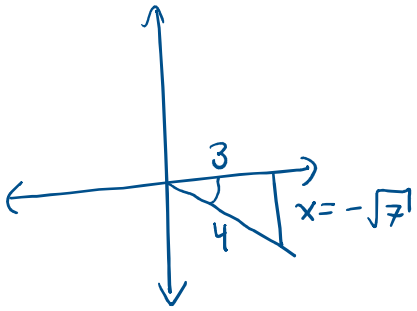
$\cos\theta = \frac{-\sqrt{75}}{10}$ $\quad \csc\theta = \frac{-10}{5}$ $\quad \cot\theta = \frac{\sqrt{75}}{5}$

$\tan\theta = \frac{-5}{-\sqrt{75}} = \frac{5}{\sqrt{75}}$ $\quad \sec\theta = \frac{-10}{\sqrt{75}}$

Ex 3: Find the exact value of the remaining six trig functions for an acute angle θ in the given quadrant.

$\cos\theta = \frac{3}{4}$ with θ in Quadrant IV

$\cos\theta = \frac{3}{4} = \frac{adj}{hyp}$



$\cos\theta = \frac{3}{4} = \frac{adj}{hyp}$

Find x w/ Pythagorean thm

$x^2 + 3^2 = 4^2$

$x^2 + 9 = 16$

$x^2 = 7$

$x = \pm\sqrt{7}$

↳ neg b/c below x-axis

$\sin\theta = \frac{opp}{hyp} = \frac{-\sqrt{7}}{4}$ $\quad \csc\theta = \frac{-4}{\sqrt{7}}$

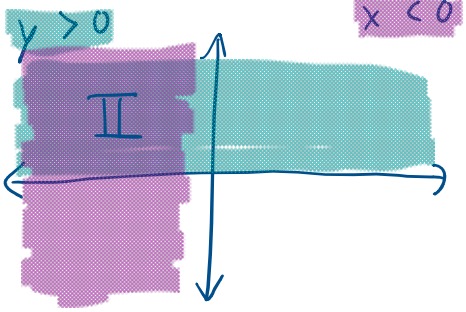
$\cos\theta = \frac{3}{4}$ $\quad \sec\theta = \frac{4}{3}$

$\tan\theta = \frac{-\sqrt{7}}{3}$ $\quad \cot\theta = -\frac{3}{\sqrt{7}}$

Ex 4: Determine the quadrant in which the terminal side of angle θ would lie, given the following conditions:

Unit Circle $(\cos\theta, \sin\theta)$

(a) $\sin\theta > 0$ and $\cos\theta < 0$



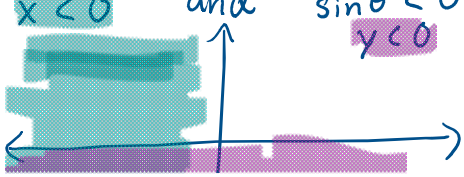
Quadrant II

(b) $\sec\theta < 0$ and $\tan\theta > 0$

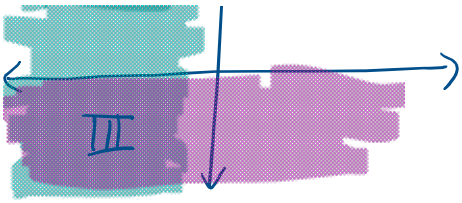
$\frac{1}{\cos\theta} < 0$ and $\frac{\sin\theta}{\cos\theta} > 0$

$\cos\theta < 0$ and $\frac{-}{-} > 0$

and $\sin\theta < 0$



Quadrant III



Quadrant III