Lesson 28: Trigonometric Eqns Pt 1

Zero Product Property

If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

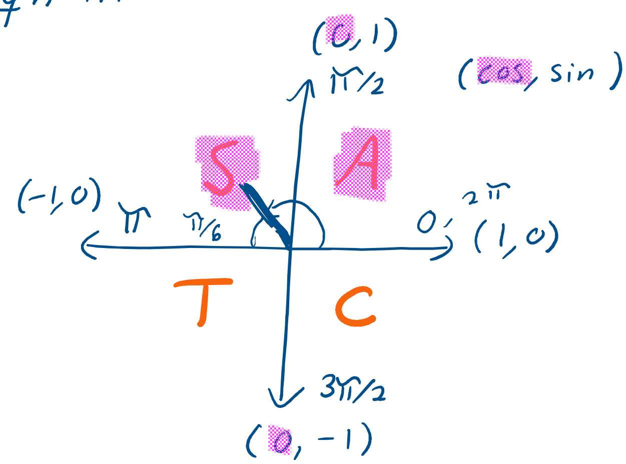
ex. $(x+1)(x+2) = 0$

$x + 1 = 0 \quad | \quad x + 2 = 0$

$x = -1 \quad | \quad x = -2$

$x = -1, -2 \quad [x = -1 \text{ or } x = -2]$

Ex 1: Find all solutions to the given eqn in the interval $[0, 2\pi)$ (Give exact solutions)



(a) $2\cos(x)\sin(x) - \cos(x) = 0$

$\cos(x)[2\sin(x) - 1] = 0$

$\cos(x) = 0 \quad | \quad 2\sin(x) - 1 = 0$

$\quad | \quad \sin(x) = \frac{1}{2}$

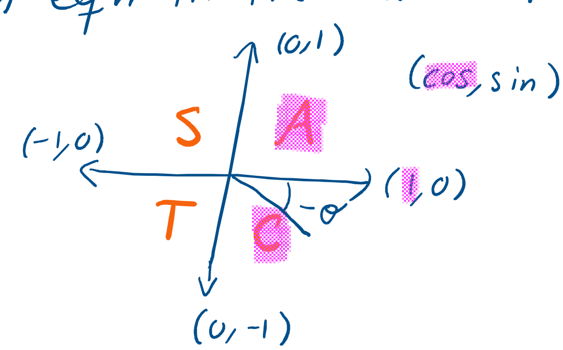
$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad | \quad x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

Trig polynomials, you can use substitution to make first steps easiers.

Ex 2: Find all solutions to the given eqn in the interval $[0, 2\pi)$. (Give exact solutions.



(a) $2\cos^2 x - 3\cos x + 1 = 0$

Let $u = \cos x$

$2u^2 - 3u + 1 = 0 \qquad 2$

Let $u = \cos x$

$2u^2 - 3u + 1 = 0$

$2u^2 - 2u - u + 1 = 0$

$2u(u-1) - (u-1) = 0$

$(2u-1)(u-1) = 0$

$\begin{array}{c|c} 2u-1=0 & u-1=0 \\ u = \frac{1}{2} & u=1 \\ \cos x = \frac{1}{2} & \cos x = 1 \\ x = \frac{\pi}{3}, -\frac{\pi}{3} & x = 0 \; ✓ \end{array}$

2
$\wedge$
$-1 \; -2$

↓
$(0,-1)$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

↓
BUT I need to write it as something in $[0, 2\pi)$

$x = \frac{\pi}{3}, \underbrace{2\pi - \frac{\pi}{3}}_{\frac{5\pi}{3}}$

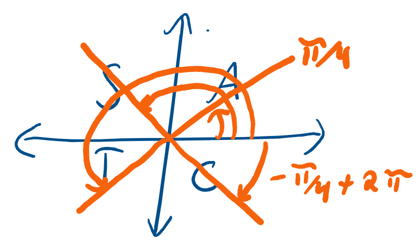
ⓑ $\tan^3(x) = \tan(x)$

$\tan^3(x) - \tan(x) = 0$

$\tan(x)\left[\tan^2(x) - 1\right] = 0$

$\begin{array}{c|l} \tan(x) = 0 & \tan^2(x) - 1 = 0 \\ x = 0 & \tan^2(x) = 1 \\ & \tan(x) = \pm\sqrt{1} = \pm 1 \\ & x = \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4} \\ & x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \end{array}$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undef



ⓒ $\underbrace{4\sin(x)\cos(x) - 2\cos(x)} + \underbrace{2\sqrt{3}\sin(x) - \sqrt{3}} = 0$

$2\cos(x)\left[2\sin(x) - 1\right] + \sqrt{3}\left[2\sin(x) - 1\right] = 0$

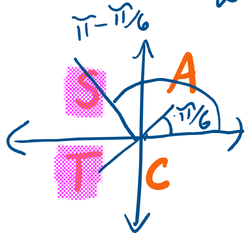
$(2\cos(x) + \sqrt{3})(2\sin(x) - 1) = 0$

$\begin{array}{c|c} 2\cos(x) + \sqrt{3} = 0 & 2\sin(x) - 1 = 0 \\ \cos(x) = -\sqrt{3} & \sin(x) = +\frac{1}{2} \end{array}$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

$2\cos(x) + \sqrt{3} = 0$

$\cos(x) = -\frac{\sqrt{3}}{2}$

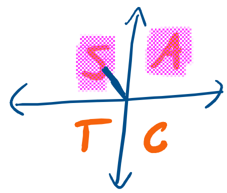


$x = \pi - \frac{\pi}{6}, \; \pi + \frac{\pi}{6}$

$= \frac{5\pi}{6}, \; \frac{7\pi}{6}$

$2\sin(x) - 1 = 0$

$\sin(x) = +\frac{1}{2}$



$x = \frac{\pi}{6}, \; \pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \; \frac{5\pi}{6}$

Sin	0	1/2	√2/2	√3/2	1
Cos	1	√3/2	√2/2	1/2	0

$x = \pi/6, \; 5\pi/6, \; 7\pi/6$

<u>Ex 3:</u> Find all solutions to the given eqn. interval $[0, 2\pi)$

ⓐ $3\cos^2(x) - 2\cos(x) - 2 = 0$

Let $\boxed{u = \cos(x)}$

$\underbrace{3u^2 - 2u - 2 = 0}_{\text{Not factorable}}$ $\quad a = 3, \; b = -2, \; c = -2$

By quadratic formula, $u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-2)}}{2(3)(-2)} = \frac{1 \pm \sqrt{7}}{3}$

$\cos x = \frac{1 \pm \sqrt{7}}{3}$

$\underbrace{\cos x = \frac{1 + \sqrt{7}}{3}}_{\text{Not possible}}$ or $\cos x = \frac{1 - \sqrt{7}}{3} \approx -0.5486$

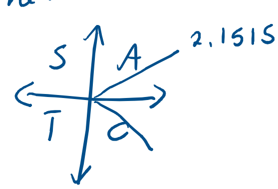
Note: $\frac{1 + \sqrt{7}}{3} \approx 1.2153$

Remember cosine range is $[-1, 1]$

$x = \cos^{-1}\left(\frac{1 - \sqrt{7}}{3}\right)$

≈ 2.1515

Another solution

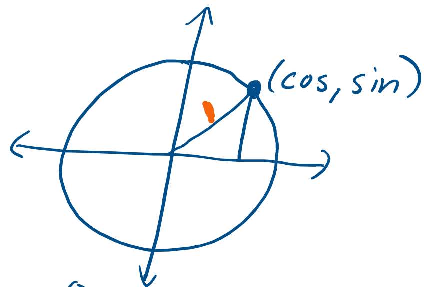


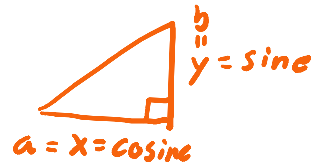
$x = 2\pi - 2.1515$

$x \approx 2\pi - 2.1515$
≈ 4.1317

Main Pythagorean identity

$$a^2+b^2=c^2$$





(1) $\sin^2\theta + \cos^2\theta = 1$

(2) Divide by $\cos^2\theta$ on both sides (1)

$$\tan^2\theta + 1 = \sec^2\theta$$

(3) Divide by $\sin^2\theta$ on both sides (1)

$$1 + \cot^2\theta = \csc^2\theta$$

Ex 4: Find all solutions to the given eqn in interval $[0, 2\pi)$

(a) $\underbrace{\sin^2(x)} - \cos^2(x) - 1 = 0$

$\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

$$\cancel{1} - \cos^2 x - \cos^2(x) \cancel{-1} = 0$$
$$-2\cos^2 x = 0$$
$$\cos^2 x = \frac{0}{-2} = 0$$
$$\cos x = \sqrt{0} = 0$$
$$\cos x = 0$$
$$x = \pi/2, 3\pi/2$$