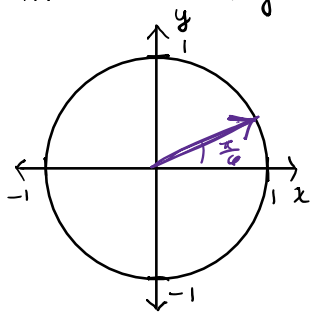
Lesson 29 - Trigonometric Equations II

General Solutions of Trigonometric Equations

- two angles with the same terminal ray (coterminal) have the same sin, cos, tan, csc, sec, cot

ex Write down a general formula for all angles coterminal to $\frac{\pi}{6}$. ($n=0$)



same terminal ray but bigger angle: $+2\pi$

$\frac{\pi}{6} + 2\pi = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$ ($n=1$)

same terminal ray but neg: $\frac{\pi}{6} - 2\pi$ ($n=-1$)

all coterminal angles: $\frac{\pi}{6} + 2\pi n$ ← n is an integer $(n = \{0, 1, 2, \ldots\ -1, -2, \ldots\})$

- θ_1 and θ_2 are coterminal if $\theta_2 = \theta_1 + 2\pi n$, where n is an integer

ex Find the general solution to the following equations. Give your answers as a number plus a multiple of π. Use the smallest possible nonnegative answers in $[0, 2\pi)$

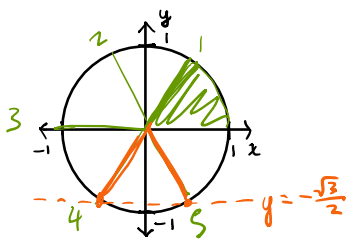
① find solutions in $[0, 2\pi)$

② write down general solution

$\sin(\theta) = -\frac{\sqrt{3}}{2}$

II

y



① $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

② $\theta = \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

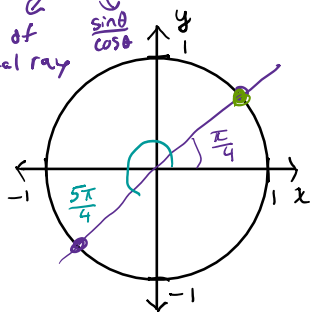
- if the terminal rays of two solutions lie on the same line, we can combine their general solutions → choose smaller angle → add πn instead of $2\pi n$

πn: half rotation; $2\pi n$: full rotation

ex Find the general solution to the following equations. Give your answers as a number plus a multiple of π. Use the smallest possible nonnegative answers in $[0, 2\pi)$

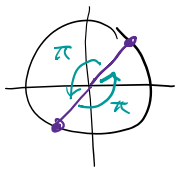
(a) $\tan(\theta) = 1$

slope of terminal ray; $\frac{\sin\theta}{\cos\theta}$



could write $\frac{\pi}{4} + 2\pi n$ and $\frac{5\pi}{4} + 2\pi n$

$\theta = \frac{\pi}{4} + \pi n$

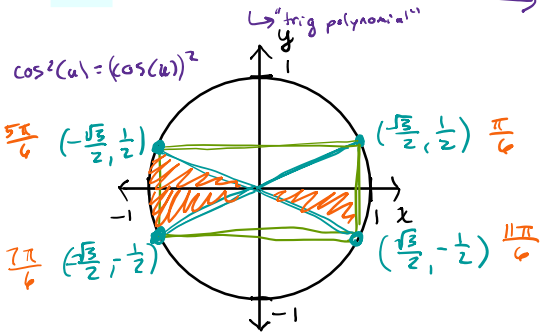


$n=0$ $\frac{\pi}{4}$

$n=1$ $\frac{\pi}{4} + \pi = \frac{\pi}{4} + \frac{4\pi}{4} = \frac{5\pi}{4}$

$n=2$ $\frac{\pi}{4} + 2\pi$

(b) $4\cos^2(u) - 3 = 0$ → "trig polynomial" → turn into polyn. → $t = \cos(u)$



$4t^2 - 3 = 0$

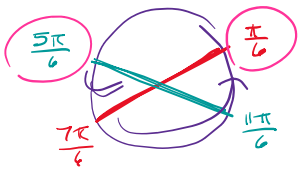
$+3 \quad +3$

$\frac{4t^2}{4} = \frac{3}{4}$

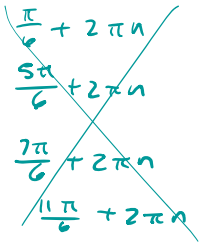
$t^2 = \frac{3}{4}$

$t = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$

back to trig → $\cos(u) = \pm\frac{\sqrt{3}}{2}$ (x-coord)



$u = \frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n$

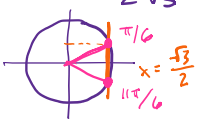


ex Find the general solution to the following equations. Give your answers as a number plus a multiple of π. Use the smallest possible nonnegative answers in $[0, 2\pi)$

(a) $\sec(\theta) = \frac{2-\sqrt{3}}{3}$

$\frac{1}{\cos(\theta)} = \frac{2\sqrt{3}}{3}$

$\cos(\theta) = \frac{3}{2\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{2\cdot 3} = \frac{\sqrt{3}}{2}$

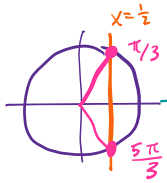


→ not on the same line, so $\theta = \frac{\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$

(b) $2\cos(t)-1=0$

$2\cos(t)=1$

$\cos(t)=\frac{1}{2}$

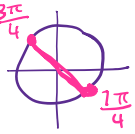


→ not on the same line,

so $\boxed{t=\frac{\pi}{3}+2\pi n,\ \frac{5\pi}{3}+2\pi n}$

(c) $\tan(\theta)+1=0$

$\tan(\theta)=-1$ → slope $=-1$ →

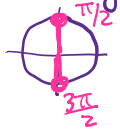


on the same line!

so $\boxed{\theta=\frac{3\pi}{4}+\pi n}$

(d) $\sin^2(\theta)=1$ $\quad(y=\sin\theta \to y^2=1 \to y=\pm\sqrt{1}=\pm 1)$

$\sin(\theta)=\pm 1$



on the same line!

so $\boxed{\theta=\frac{\pi}{2}+\pi n}$

(e) $\tan^2(t)+\tan(t)=0$

$x=\tan(t)$ ↓

$x^2+x=0$

$x(x+1)=0$

↓ ↓

$x=0$ or $x=-1$

slope $=0$

$\tan\theta=0$

$\tan\theta=-1$ → from (c),

on the same line!

so $\boxed{\theta=0+\pi n}$

$\boxed{\theta=\frac{3\pi}{4}+\pi n}$

[CHALLENGE: answer the remaining using only one equation per problem!]

(f) $\sec^2(t)-2=0$

$x=\sec(t)$ ↓

$x^2-2=0$

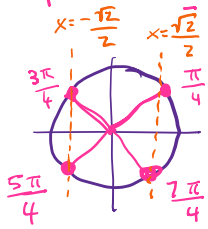
$x^2=2$

$x=\pm\sqrt{2}$

$\sec(t)=\pm\sqrt{2}$

$\frac{1}{\cos(t)}=\pm\sqrt{2}$

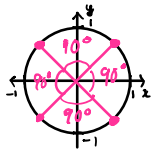
$\cos(t)=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}$



two lines! →

$t=\frac{\pi}{4}+\pi n,\ \frac{3\pi}{4}+\pi n$ ...?

can simplify more:



← all solns. are equally spaced $(\frac{\pi}{2})$

so: $\boxed{t=\frac{\pi}{4}+\frac{\pi}{2}n}$

(g) $\csc^2(x)=2+\csc(x)$

$t=\csc(x)$ ↓

$x^2=2+x$

$x^2-x-2=0$

$(x-2)(x+1)=0$

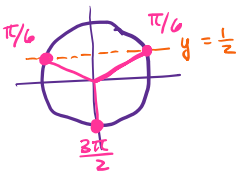
↓ ↓

$x=2$ $\quad x=-1$

$\csc(x)=2 \to \sin(x)=\frac{1}{2}$

or

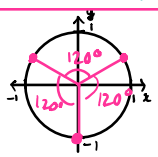
$\csc(x)=-1 \to \sin(x)=-1$



no two solutions on the same line.

$x=\frac{\pi}{6}+2\pi n,\ \frac{5\pi}{6}+2\pi n,\ \frac{3\pi}{2}+2\pi$ ...?

can simplify more:



← all solns. are equally spaced $(\frac{2\pi}{3})$

so: $\boxed{x=\frac{\pi}{6}+\frac{2\pi}{3}n}$