Wednesday, January 14, 2026 11:28 AM

Lesson 2: Simplifying Algebraic Expressions I

Review of Polynomials

A polynomial is of the form

$$\underbrace{a_n x^{n}}_{\text{leading term}} + a_{n-1} x^{n-1} + \dots + \underbrace{a_2 x^2}_{\text{quadratic term}} + \underbrace{a_1 x^1}_{\text{linear term}} + \underbrace{a_0 x^0}_{\text{constant term}}$$

$n \rightarrow$ degree

Standard form (exponents are decreasing

for a whole number $n \geq 0$,

Definitions:

- coefficients a_i where i is a positive integer
- term:
  - ↳ leading term ($a_n x^n$)
  - ↳ quadratic term ($a_2 x^2$)
  - ↳ linear term ($a_1 x^1$)
  - ↳ constant term (a_0)
- degree (highest power of the variable w/ non-zero coefficient)
- Special types of polynomials
  - ↳ monomial (one term)
  - ↳ binomial (2 terms)
  - ↳ trinomial (3 terms)
- different degrees
  - ↳ constant
  - ↳ linear
  - ↳ quadratic
  - ↳ cubic

Facts: ① The sum or difference of polynomials is also polynomial
② The product of polynomials is also a polynomial.

Ex 1: Sum the following polynomials $x^2 + 2x + 1$ and $x^3 + x + 1$

$$x^2 + 2x + 1 + x^3 + x + 1$$
$$= x^3 + x^2 + 2x + x + 1 + 1$$

$$\begin{array}{r} x^2 + 2x + 1 \\ + \; x^3 + + x + 1 \\ \hline \end{array}$$

$= x^3 + x^2 + 2x + x + 1 + 1$

$= x^3 + x^2 + 3x + 2$

$$\begin{array}{r} + \; x^3 + \quad + x + 1 \\ \hline x^3 + x^2 + 3x + 2 \end{array}$$

Ex 2: Find the product of (x^2+1) and $(2x-5)$

Way 1: Distribute $(x^2+1)(2x+5) = x^2(2x+5) + 1(2x+5)$

$= 2x^3 + 5x^2 + 2x + 5$

Way 2: Table

x^2+1 across the top; $2x+5$ down the side:

	x^2	1
$2x$	$2x \cdot x^2 = 2x^3$	$2x \cdot 1 = 2x$
$+5$	$5 \cdot x^2 = 5x^2$	$5 \cdot 1 = 5$

Answer: $2x^3 + 5x^2 + 2x + 5$

The process we just did is called expanding (or multiplying out, or distributing, etc.) Expanding can be one way of simplifying.

Why? Ex 3: Simplify $(x-y)(x^2+xy+y^2) = x^3 \underbrace{- x^2y + x^2y}_{\text{eliminate}} \underbrace{- xy^2 + xy^2}_{\text{eliminate}} - y^3$

	x^2	xy	y^2
x	x^3	x^2y	xy^2
$-y$	$-x^2y$	$-xy^2$	$-y^3$

$= x^3 - y^3$

- GCF (Greatest common factor)

  The greatest common factor of the terms of a polynomial is the "largest" (largest coefficient, highest degree) monomial that divides all the terms.

  [We can also find GCFs of expressions not in standard forms]

Ex 4: Find the GCF of the following

ⓐ $36x^3y^2 - 18x^2y^3 + 30x^5y$

$= 6x^2y\left[6xy - 3y^2 + 5x^3\right]$

36 ∧ 6 6   18 ∧ 3 6   30 ∧ 5 6

GCF: $6x^2y$

ⓑ [non-standard form] $4(x+2)^6(x+5)^1 - 20(x+2)^3(x+6)(x+5)^1$

$= 4(x+2)^3(x+5)\left[(x+2)^3 - 5(x+6)\right]$

GCF: $4(x+2)^3(x+5)$

Factoring is the process of breaking up a polynomial into the product of two or more polynomials of lower degree.

(Factoring is the opposite process of expanding)

expand

$$(x^2+1)(2x+5) = 2x^3 + 5x^2 + 2x + 5$$

factor

Before trying to factor, find GCF and factor that first.

① Factoring by grouping

Sometimes we can group the terms of a polynomial to factor it. To be successful, each group must have the same # of terms.

Ex 5: Factor $\underbrace{x^6 + 3x^5} + \underbrace{2x^2 + 6x}$

$= x^5\underbrace{(x+3)} + 2x\underbrace{(x+3)}$

$= (x+3)(x^5 + 2x)$

$$= (x+3)(x^5+2x)$$
$$= (x+3)(x)(x^4+2)$$

Ex 6: Factor $\underbrace{2x^2-3x}+\underbrace{2x-3}$

$$= x\underbrace{(2x-3)}+1\underbrace{(2x-3)}$$
$$= (x+1)(2x-3)$$

② Factoring quadratics

(Note: Sometimes factoring is possible, but sometimes it is not. [If not, the polynomial is irreducible])

ⓐ Factoring quadratics w/ no linear term

We can use the fact that $a^2-b^2=(a+b)(a-b)$

Ex 7: Factor $9x^2-25$

$$= (\underset{a}{3x})^2-(\underset{b}{5})^2 = (3x+5)(3x-5)$$

Ex 8: Factor $16j^4-121$

$$= (\underset{a}{4j^2})^2-(\underset{b}{11})^2 = (4j^2+11)(4j^2-11)$$

ⓑ Factoring quadratics w/ a linear term.

$$ax^2+bx+c$$

The method is the ac-method.

Ex 9: Factor: $x^2-5x-14$ $\left.\begin{matrix} \\ ax^2+bx+c\end{matrix}\right\}$ $a=1$ $b=-5$ $c=-14$

$a\cdot c = 1\cdot -14 = -14$

$-13 = 1-14$

$-5 = 2-7$

$-5 = 2 - 7$

$$x^2 - 5x - 14 = \underbrace{x^2 + 2x} \underbrace{- 7x - 14}$$

$$= x\underbrace{(x+2)} - 7\underbrace{(x+2)}$$

$$= (x-7)(x+2)$$

Ex 10: Factor $6t^2 - 11t - 35$ $\quad a = 6,\ b = -11,\ c = -35$

$at^2 + bt + c$