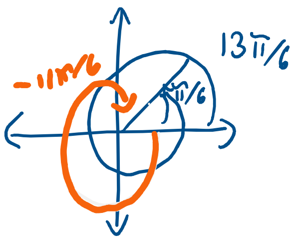
Monday, April 13, 2026 11:32 AM

Lesson 29+30: Trigonometric Eqns II and III



Write down a general formula for all angles coterminal to $\frac{\pi}{6}$

$$\frac{\pi}{6} + 2\pi = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$$

same is true if I when the negative way

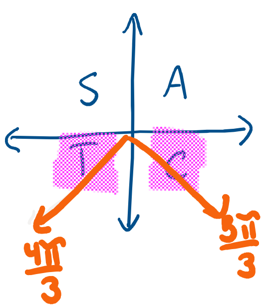
$$\frac{\pi}{6} - 2\pi = \frac{\pi}{6} - \frac{12\pi}{6} = -\frac{11\pi}{6}$$

All coterminal angles: $\frac{\pi}{6} + 2\pi n$ where n is integer

Ex 1: Find the general solution to the following eqns. Give your answers as a number plus a multiple of π.

$\sin(\theta) = -\frac{\sqrt{3}}{2}$ By the Table, $\sin\theta = \frac{\sqrt{3}}{2}$ @ $\theta = \frac{\pi}{3}$ ↗ reference angle

Q3: $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$ Q4: $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$



$\theta = \frac{4\pi}{3} + 2\pi n$ $\theta = \frac{5\pi}{3} + 2\pi n$

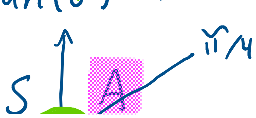
where n is integer

If the terminal rays of two solutions lie on the same line, we can combine their general solutions.

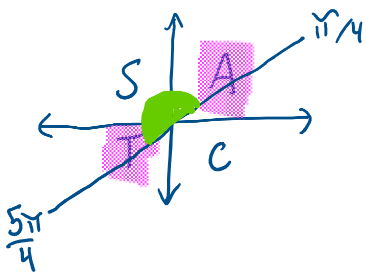
① choose smaller angle
② add πn instead $2\pi n$

Ex 2: Find the general solution to the following eqns. Give your answers as a number plus a multiple of π.

$\tan(\theta) = 1 \longrightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$



$\theta = \frac{\pi}{4} + \pi n$



$\theta = \frac{\pi}{4} + \pi n$

Ex 3: Find the general solution to the following equations. Give your answers as the smallest solution greater than 0 plus a multiple of π.

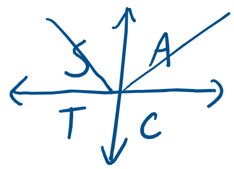
ⓐ $\sin(2x) = \frac{1}{2}$

Let $\theta = 2x$

$\sin\theta = \frac{1}{2} \Rightarrow$ By the table is $\theta = \frac{\pi}{6}$

also

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$



General solution

$\theta = \frac{\pi}{6} + 2\pi n$

$\theta = \frac{5\pi}{6} + 2\pi n$

Substitute back

$$\frac{2x}{2} = \frac{\frac{\pi}{6} + 2\pi n}{2}$$

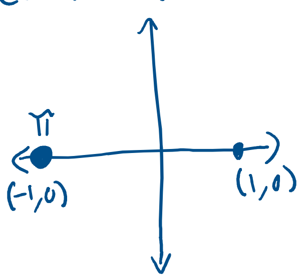
$$x = \frac{\pi}{12} + \pi n$$

$$\frac{2x}{2} = \frac{\frac{5\pi}{6} + 2\pi n}{2}$$

$$x = \frac{5\pi}{12} + \pi n$$

$$\frac{\frac{\pi}{6}}{\frac{2}{1}} = \frac{\pi}{6} \cdot \frac{1}{2} = \frac{\pi}{12}$$

ⓑ $\cos(4\omega) = -1$



Let $\theta = 4\omega$

$\cos(\theta) = -1 \Rightarrow \theta = \pi + 2\pi n$

Substitute back

$$4\omega = \pi + 2\pi n$$

$$\omega = \frac{\pi}{4} + \frac{2\pi n}{4}$$

$$= \frac{\pi}{4} + \frac{\pi n}{2}$$

ⓒ $2\cos(3\pi\theta) + 1 = 0$

$\qquad\qquad -1 \quad -1$

$$\frac{2\cos(3\pi\theta)}{2} = \frac{-1}{2}$$

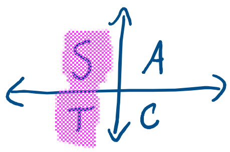
$$\frac{2\cos(3\pi\theta)}{2} \qquad \frac{\quad}{2}$$

$$\cos(3\pi\theta) = -\frac{1}{2}$$

Let $x = 3\pi\theta$

$\cos(x) = -\frac{1}{2} \Rightarrow \quad \cos(x) = \frac{1}{2}$ @ $x = \pi/3$

But $-$,



Q2: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} + 2\pi n$

Q3: $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} + 2\pi n$

Substitute back $x = 3\pi\theta$

$$\frac{3\pi\theta}{3\pi} = \frac{\frac{2\cancel{\pi}}{3} + 2\cancel{\pi} n}{3\cancel{\pi}} \qquad\qquad \frac{3\pi\theta}{3\pi} = \frac{\frac{4\cancel{\pi}}{3} + 2\cancel{\pi} n}{3\cancel{\pi}}$$

$$\theta = \frac{2}{9} + \frac{2}{3}n \qquad\qquad \theta = \frac{4}{9} + \frac{2}{3}n$$

ⓓ $2\cos^2(3t) + \cos(3t) = 1$

$2\cos^2(3t) + \cos(3t) - 1 = 0$

$u = \cos(3t)$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2} \qquad u = -1$

But $u = \cos(3t)$

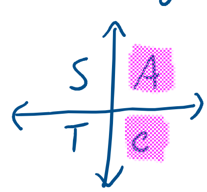
$\cos(3t) = \frac{1}{2}$ or $\cos(3t) = -1$

$\cos(3t) = \frac{1}{2}$

$\theta = 3t$

$\cos\theta = \frac{1}{2}$

By Table $\theta = \pi/3 + 2\pi n$



$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} + 2\pi n$

$\cos(3t) = -1$

$\theta = 3t$

$\cos\theta = -1$

$\theta = \pi + 2\pi n$

$$\frac{3t}{3} = \theta = \frac{\frac{\pi}{3} + 2\pi n}{3} \qquad \frac{3t}{3} = \theta = \frac{\frac{5\pi}{3} + 2\pi n}{3} \qquad \frac{3t}{3} = \theta = \frac{\pi + 2\pi n}{3}$$

$$t = \frac{\pi}{9} + \frac{2\pi n}{3} \qquad \theta = \frac{5\pi}{9} + \frac{2\pi n}{3} \qquad \theta = \frac{\pi}{3} + \frac{2\pi n}{3}$$

where n is integer

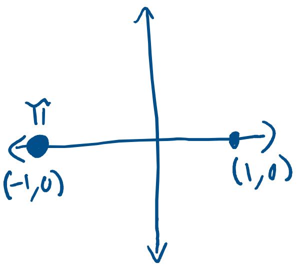
Ex 4: Find the exact value of all solutions in the interval $\underbrace{[0, 2\pi)}_{0,\ 8\pi}$.

ⓐ $\cos(4w) = -1$

Ex 4. Find the exact valu

$0, \frac{8\pi}{4}$

(a) $\cos(4\omega) = -1$



(cos, sin)

Let $\theta = 4\omega$

$\cos(\theta) = -1 \Rightarrow \theta = \pi + 2\pi n$

Substitute back

$4\omega = \pi + 2\pi n$

$\omega = \boxed{\frac{\pi}{4} + \frac{2\pi n}{4}}$

$= \frac{\pi}{4} + \frac{\pi n}{2}$ where n is integer

$n=0$: $\omega = \frac{\pi}{4} + \frac{2\pi(0)}{4} = \frac{\pi}{4}$

$n=1$: $\omega = \frac{\pi}{4} + \frac{2\pi(1)}{4} = \frac{3\pi}{4}$

$n=2$: $\omega = \frac{\pi}{4} + \frac{2\pi(2)}{4} = \frac{5\pi}{4}$

$n=3$: $\omega = \frac{\pi}{4} + \frac{2\pi(3)}{4} = \frac{7\pi}{4}$ Done!

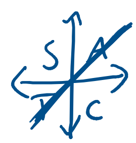
(b) $\sqrt{3}\cot\left(\frac{1}{2}t\right) = 1$

$\cot\left(\frac{1}{2}t\right) = \frac{1}{\sqrt{3}}$

$\frac{1}{\tan\left(\frac{1}{2}t\right)} = \frac{1}{\sqrt{3}}$

$\tan\left(\frac{1}{2}t\right) = \sqrt{3}$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined



Let $\theta = \frac{1}{2}t$

$\tan\theta = \sqrt{3} \Rightarrow$ By Table $\theta = \frac{\pi}{3} + \pi n$

Substitute back $\theta = \frac{1}{2}t$

$\frac{1}{2}t = \frac{\pi}{3} + \pi n$

$t = 2\left(\frac{\pi}{3} + \pi n\right) = \frac{2\pi}{3} + 2\pi n$