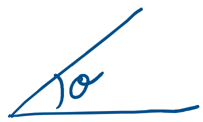
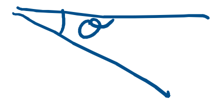
Lesson 31: Applications of Trigonometry

Angle of elevation/inclination



(angle above horizon)

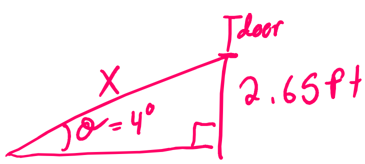
Angle of depression



(angle below horizon)

Ex 1: In the design of a new building, a doorway is 2.65 ft above the ground. A ramp, at an angle of 4° with the ground, is to be built up to the doorway. How long will the ramp be?

angle of elevation



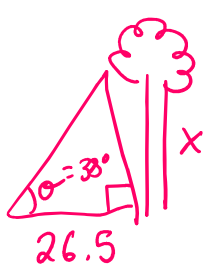
$$\sin(\theta) = \frac{2.65}{x}$$

$$\sin(4^\circ) = \frac{2.65}{x}$$

$$\frac{x\sin(4^\circ)}{\sin(4^\circ)} = \frac{2.65}{\sin(4^\circ)}$$

$$x = \frac{2.65}{\sin(4^\circ)} = 37.989 \text{ ft}$$

Ex 2: When the elevation of the sun is 38°, a tree casts a shadow 26.5 ft long. How tall is the tree?



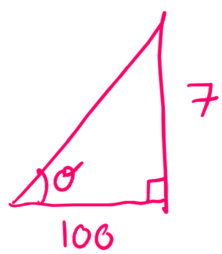
$$\tan\theta = \frac{x}{26.5}$$

$$\tan(38^\circ) = \frac{x}{26.5}$$

$$26.5\tan(38^\circ) = x$$

$$x = 20.7 \text{ ft}$$

Ex 3: The angle of inclination of a road is often expressed as percent grade, which is the vertical rise divided by the horizontal percent grade, which is the vertical rise divided by the horizontal run (expressed as a percent. A 7% grade corresponds to a road that rises 7 ft for every 100 ft along the horizontal. Find the angle of inclination that corresponds to 7% grade.



$$\tan\theta = \frac{7}{100}$$

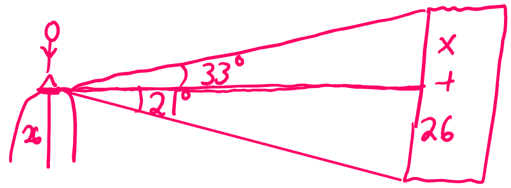
$$\theta = \underbrace{\tan^{-1}\left(\frac{7}{100}\right)}_{\text{radians}} = 4.0042^\circ$$

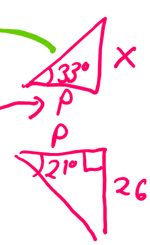
radians → convert to degrees

(answer × $\frac{180^\circ}{\pi}$)

Ex 4: A surveyor standing on a hill 26 ft high looks at a building across a river. The surveyor determines that the angle of depression to the base of the building is 21° and the angle of elevation to the top of the building to be 33°. Calculate the height of the building in ft.

Find p to find x.





$$\Rightarrow \tan(21^\circ) = \frac{26}{p}$$

$$p\tan(21^\circ) = 26$$

$$p = \frac{26}{\tan(21^\circ)}$$

$$\tan(33^\circ) = \frac{x}{p}$$

$$p \cdot \tan(33^\circ) = x$$

$$\frac{26}{\tan(21^\circ)} \cdot \tan(33^\circ) = x$$

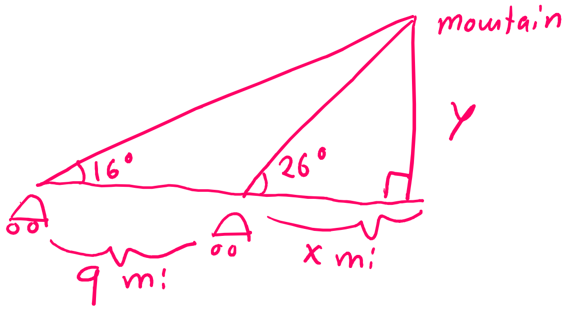
$$x = 43.986 \text{ ft}$$

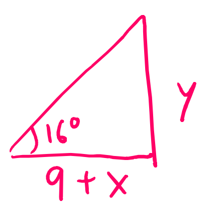
$x = 43.986$ ft

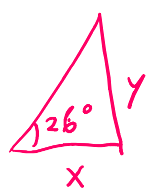
Ex 6: A driver in a car traveling along a straight level road at 54 miles per hour sees a mountain in the distance. The driver determines that the angle of elevation to the top of the mountain is 16°. After 10 minutes, the driver determines that the angle of elevation to the top of the mountain is 26°. Calculate the height of the mountain.

$$54 \frac{\text{miles}}{\text{hr}} \cdot 10 \cancel{\text{min}}$$

$$54 \frac{\text{miles}}{\cancel{\text{hr}}} \quad \frac{10}{60} \cancel{\text{hr}} = 9 \text{ miles}$$







$$\tan(16°) = \frac{y}{9+x}$$

$$\tan(26°) = \frac{y}{x}$$

Solve for x

$$x\tan(26°) = y$$

$$x = \frac{y}{\tan(26°)}$$

$$\frac{\tan(16°)}{1} = \frac{y}{9 + \frac{y}{\tan(26°)}}$$

$$9\tan(16°) + \frac{y\tan(16°)}{\tan(26°)} = y$$

$$9\tan(16^\circ) \cdot \frac{\;}{\tan(26^\circ)} = y$$

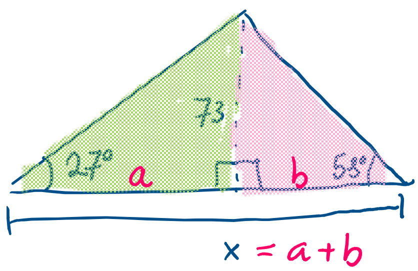
$$9\tan(16^\circ) = y - y\frac{\tan(16^\circ)}{\tan(26^\circ)}$$

$$= y\left(1 - \frac{\tan(16^\circ)}{\tan(26^\circ)}\right)$$

$$y = \frac{9\tan(16^\circ)}{1 - \frac{\tan(16^\circ)}{\tan(26^\circ)}} = 33{,}066 \text{ ft}$$

Non-right triangles

Ex 7: Find x.



$$\tan(27^\circ) = \frac{73}{a}$$

$$a\tan(27^\circ) = 73$$

$$a = \frac{73}{\tan(27^\circ)}$$

$$\tan(58^\circ) = \frac{73}{b}$$

$$b\tan(58^\circ) = 73$$

$$b = \frac{73}{\tan(58^\circ)}$$

add them

$x = 188.386$