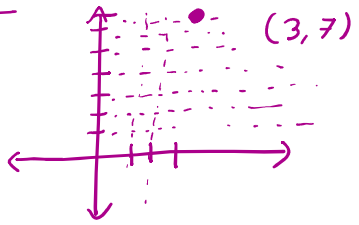
Lesson 32: Polar Coordinates, Equations, and Graphs

Polar Coordinates



- Coordinates: instructions for how to find a point
- Cartesian/rectangular coordinates: $P(x,y)$
  - ↳ special point: the origin $O(0,0)$

$P(x,y)$ { x tells us to go right or left from O & by how much
y tells us to go up or down from O & by how much

- Polar coordinates: $P(r,\theta)$ (r: radius, θ: angle)


$P(r,\theta)$ { θ tells us to rotate counterclockwise (+) or clockwise (−) & by how much
r tells us to go forward or backward from O & by how much

Polar grid

$r \quad \theta$

$(4, \frac{2\pi}{3})$

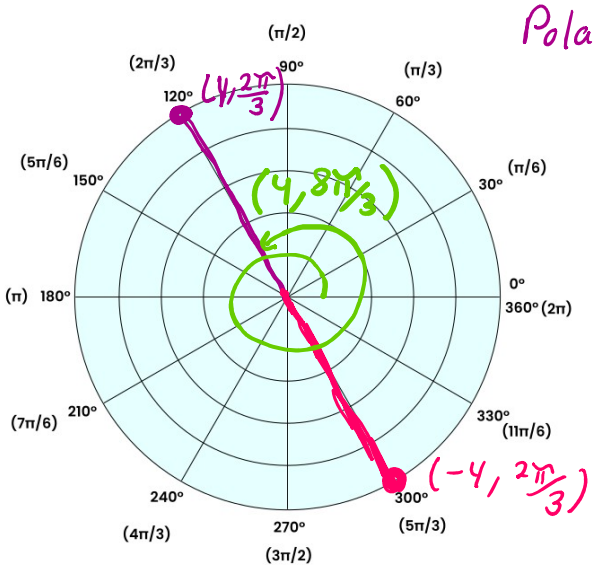
$(-4, \frac{2\pi}{3})$

$(4, \frac{8\pi}{3})$

$\downarrow$

$\frac{8\pi}{3} - 2\pi = \frac{8\pi}{3} - \frac{6\pi}{3} = \frac{2\pi}{3}$

Polar grid



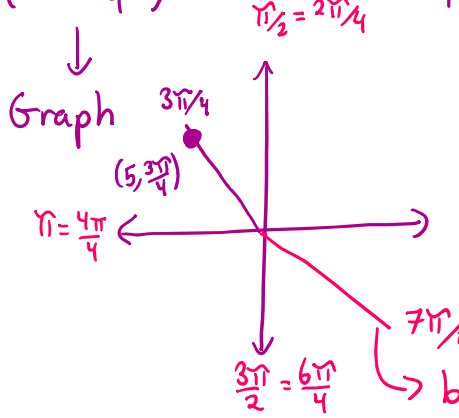
Ex 1: Do the following pairs of coordinates represent the same point?

(a) $(5, \frac{3\pi}{4})$ and $(-5, \frac{7\pi}{4})$ Yes

$\pi/2 = 2\pi/4$

(a) $(5, \frac{-11}{4})$ and $(5, \frac{1}{4})$ Yes

↓ Graph



→ but I have −5 so in the opposite way

(b) $(3, \frac{2\pi}{3})$ and $(-3, \frac{-5\pi}{3})$ No

First check that the x-coordinates are opposite signs,
If that's true and both angles are positive then subtract
and if it equals π, then yes.
If the angles are different signs
Check if $\theta_1 - \theta_2 = \pi + 2\pi n$

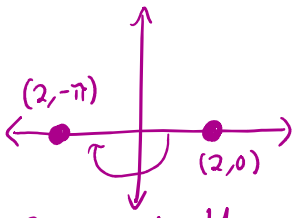
$$\frac{2\pi}{3} - \left(-\frac{5\pi}{3}\right) = \frac{2\pi}{3} + \frac{5\pi}{3} = \frac{7\pi}{3}$$

$$= \frac{3\pi}{3} + \frac{4\pi}{3}$$

$$= \pi + \left(\frac{4}{3}\right)\pi$$

→ Is it an integer? No.

(c) $(2, 0)$ and $(2, -\pi)$ No b/c not same spot.

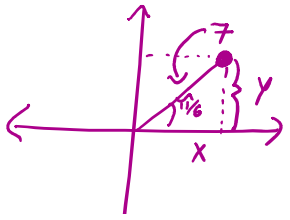


Ex 2: Convert the given polar coordinates to cartesian coordinates

Unit circle ⇒ $(\cos\theta, \sin\theta)$

Any circle ⇒ $(r\cos\theta, r\sin\theta)$ ☆

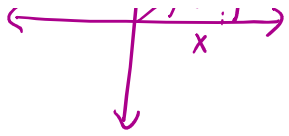
(a) $(7, \frac{\pi}{6})$
↓ ↓
r θ



$$\Rightarrow \cos\left(\frac{\pi}{6}\right) = \frac{x}{7}$$

$$x = 7\cos\left(\frac{\pi}{6}\right) = 7\frac{\sqrt{3}}{2}$$

r θ



$x = r\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin\left(\frac{\pi}{6}\right) = \frac{y}{7}$

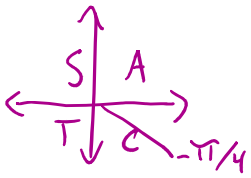
$y = 7\sin\left(\frac{\pi}{6}\right) = 7 \cdot \frac{1}{2} = \frac{7}{2}$

Answer: $\left(\frac{7\sqrt{3}}{2}, \frac{7}{2}\right)$

(b) $\left(2, -\frac{\pi}{4}\right)$

$x = r\cos\theta = 2\cos(-\pi/4) = 2\cos(\pi/4) = 2\frac{\sqrt{2}}{2} = \sqrt{2}$

$y = r\sin\theta = 2\sin(-\pi/4) = -2\sin(\pi/4) = -2\frac{\sqrt{2}}{2} = -\sqrt{2}$

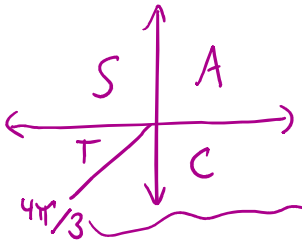


Answer: $(\sqrt{2}, -\sqrt{2})$

(c) $\left(3, \frac{4\pi}{3}\right)$

$x = r\cos\theta = 3\cos\left(\frac{4\pi}{3}\right) = -3\cos(\pi/3) = -3 \cdot \frac{1}{2} = -\frac{3}{2}$

$y = r\sin\theta = 3\sin\left(\frac{4\pi}{3}\right) = -3\sin(\pi/3) = -3 \cdot \frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2}$



reference angle $\frac{4\pi}{3} - \pi = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

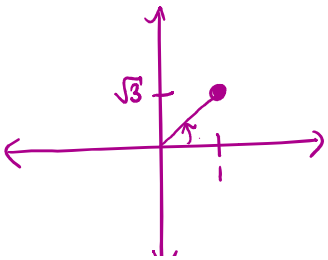
Answer: $\left(-\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$

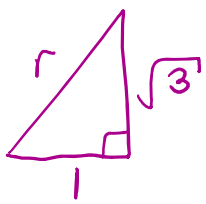
Recap: $P(r, \theta)$ in Cartesian coordinates is $(\overset{x}{\overset{\|}{r\cos\theta}}, \overset{y}{\overset{\|}{r\sin\theta}})$

Ex 3: Find the polar coordinates (r, θ) for each point.

$r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

(a) $(x, y) = (1, \sqrt{3})$





$1^2 + \sqrt{3}^2 = r^2$

$\sqrt{1^2 + (\sqrt{3})^2} = r$

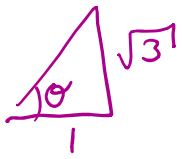
$\sqrt{1+3} = r$

$\sqrt{4} = r$

$2 = r$

$\sqrt{4} = r$

$2 = r$



$\tan\theta = \frac{\sqrt{3}}{1}$ ← y-coordinate, ← x-coordinate

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = \frac{\pi}{3}$

Answer: $\left(2, \frac{\pi}{3}\right)$

(b) $(x,y) = (4,-4)$

$r = \sqrt{x^2+y^2}$

$= \sqrt{4^2+(-4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= \sqrt{2}\cdot\sqrt{16}$

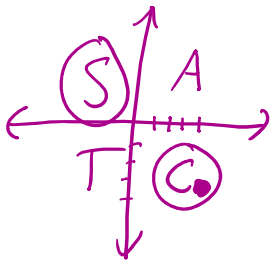
$= 4\sqrt{2}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-4}{4}$

$\tan\theta = -1$

ref $\theta = \frac{\pi}{4}$



real angle $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Answer: $\left(4\sqrt{2}, \frac{7\pi}{4}\right)$

(c) $(x,y) = (-5\sqrt{3}, 5)$

$r = \sqrt{x^2+y^2}$

$= \sqrt{(5\sqrt{3})^2+5^2}$

$= \sqrt{25\cdot 3+25}$

$= \sqrt{75+25}$

$= \sqrt{100}$

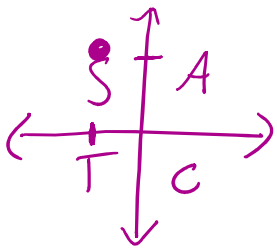
$= 10$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{5}{-5\sqrt{3}}$

$\tan\theta = \frac{-1}{\sqrt{3}}$

ref $\theta = \frac{\pi}{6}$



$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Answer: $\left(10, \frac{5\pi}{6}\right)$

Recap: P(x,y) to polar coordinates use $r = \sqrt{x^2+y^2}$

$\tan\theta = y/x$

✱ Finding θ includes working w/ reference angles.