SU22_MA16010_L1_Handout_Finding_Limits_Graphically

Friday, April 24, 2026 11:28 AM

Final Exam on 5/5 @ 8am in LO136

30 questions

↳ 15 questions from Exam 1, Exam 2, Exam 3 (numbers changed)

↳ 5 from Exam 1

↳ 5 from Exam 2

↳ 5 from Exam 3

Max.

↳ 15 questions from after Exam 3 (Lessons 27-35)

Lesson 35: Limits

Finding Limits Numerically

Def: If f(x) approaches L as x approaches c we say that the limit of f(x) as x approaches c is L.

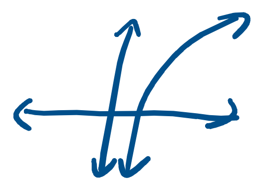
i.e. $\lim_{x \to c} f(x) = L$

Note that f does not need to be defined @ $x = c$ for the limit to exist.

Def: If f(x) is increasing or decreasing without bounds as x approaches c, then $\lim_{x \to c} f(x)$ is an infinite limit.

↳ If f(x) increasing without bound,

$\lim_{x \to c} f(x) = +\infty$



↳ If f(x) decreasing without bound,

$\lim_{x \to c} f(x) = -\infty$

lim f(x) we find it by evaluating f(x) at values of x that are

$\lim_{x \to c} f(x)$ we find it by evaluating $f(x)$ at values of x that are getting closer and closer to c and see what happens with the values of the function.

Ex 1: Evaluate $\lim_{x \to 4} (2x-3)$ numerically

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	4.8	4.98	4.998		5.002	5.02	5.2

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Recall $\lim_{x \to c} f(x)$. So $f(x) = 2x-3$

$$\lim_{x \to 4} (2x-3) = 5$$

Ex 2: Evaluate $\lim_{x \to 3} \frac{x^3 - 3x^2}{x-3}$ numerically

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	8.41	8.9401	8.994		9.006	9.0601	9.61

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Recall $\lim_{x \to c} f(x)$. So $f(x) = \frac{x^3 - 3x^2}{x-3}$

$$\lim_{x \to 3} \frac{x^3 - 3x^2}{x-3} = 9$$

Ex 3: Given $f(x) = \begin{cases} x^2+1 & \text{if } x \neq -4 \\ 2 & \text{if } x = -4 \end{cases}$ ☆

Evaluate $\lim_{x \to -4} f(x)$ numerically

x	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9
$f(x)$	17.81	17.0801	17.0080		16.992	16.9201	16.21

x	-4.1	-4.01	-4.001	-4	-3.999	[illegible]	[illegible]
$f(x)$	17.81	17.0801	17.0080		16.992	16.9201	16.21

$\longrightarrow 17 \longleftarrow$

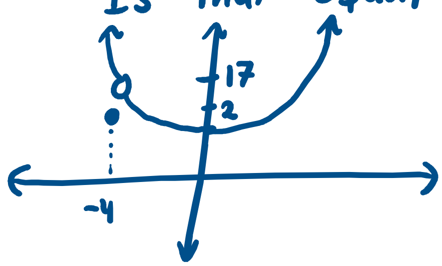
Again What is $f(x)$ when $x \neq -4$? $f(x) = x^2 + 1$

$$\lim_{x \to -4} f(x) = 17$$

Question: What is $f(-4)$? 2

Is that equal to $\lim_{x \to -4} f(x)$? No!

Why?



One-Sided Limits

Definition: A one-sided limit is the value that the function on $f(x)$ approaches L as x approaches c from the left or right.

Left-sided limit: $\lim_{x \to c^-} f(x) = L$

Right-sided limit: $\lim_{x \to c^+} f(x) = L$

Ex 4: Evaluate $\lim_{x \to -1^-} \frac{1}{(x+1)^2}$ and $\lim_{x \to -1^+} \frac{1}{(x+1)^2}$

x	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
$f(x)$	100	10000	100000		10000	1000	100

$\longrightarrow \infty$ $\qquad$ $\longleftarrow \infty$

$$\lim_{x \to -1^-} \frac{1}{(x+1)^2} = \infty \quad \text{and} \quad \lim_{x \to -1^+} \frac{1}{(x+1)^2} = \infty$$

Finding Limits Graphically

Graphically, we will look at the portion of the curve of $f(x)$ near $x=c$ and see what the function value, y, approaches as x gets closer to c from the left or right, respectively.

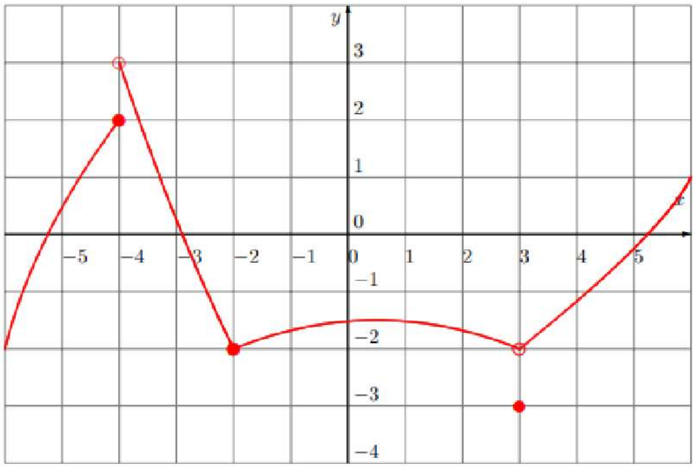
If $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$, then

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = \lim_{x \to c} f(x). \; [*]$$

Note this doesn't imply that $[*] = f(c)$

Example 3 (From Worksheet)

3. Consider the following function defined by its graph:



Find the following limits:

A) $\lim_{x\to -4^-} f(x) =$ 2

B) $\lim_{x\to -4^+} f(x) =$ 3

C) $\lim_{x\to -4} f(x) =$ DNE

D) $f(-4) =$ 2

E) $\lim_{x\to -2^-} f(x) =$ -2

F) $\lim_{x\to -2^+} f(x) =$ -2

G) $\lim_{x\to -2} f(x) =$ -2

H) $f(-2) =$ -2

I) $\lim_{x\to 3^-} f(x) =$ -2

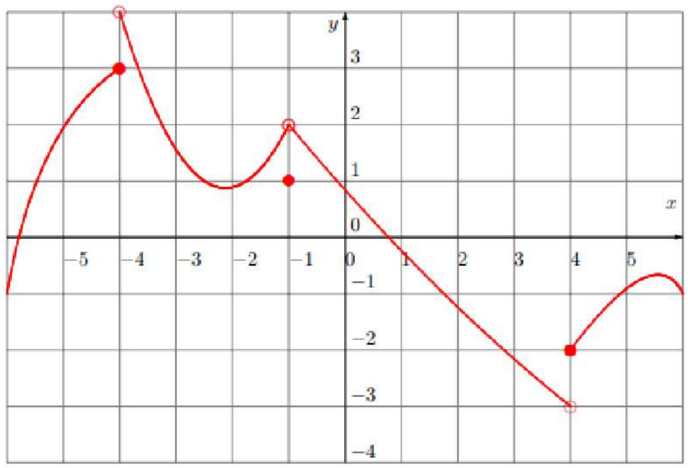
J) $\lim_{x\to 3^+} f(x) =$ -2

K) $\lim_{x\to 3} f(x) =$ -2

L) $f(3) =$ 3

Example 1 (From Worksheet)

1. Consider the following function defined by its graph:



Find the following limits:

A) $\lim_{x\to-4^-} f(x) =$ 3

B) $\lim_{x\to-4^+} f(x) =$ 4

C) $\lim_{x\to-4} f(x) =$ DNE

D) $f(-4) =$ 3

E) $\lim_{x\to-1^-} f(x) =$ 2

F) $\lim_{x\to-1^+} f(x) =$ 2

G) $\lim_{x\to-1} f(x) =$ 2

H) $f(-1) =$ 1

I) $\lim_{x\to4^-} f(x) =$ -3

J) $\lim_{x\to4^+} f(x) =$ -2

K) $\lim_{x\to4} f(x) =$ DNE

L) $f(4) =$ -2