

# Lesson 3: Simplifying Algebraic Expressions II

## Fractional Expression

- Basic operations

Ex 1: Simplify

$$\textcircled{a} \frac{1}{3} - \frac{2}{9} = \frac{1}{3} \cdot \frac{3}{3} - \frac{2}{9} = \frac{3}{9} - \frac{2}{9} = \frac{1}{9}$$

$$\textcircled{b} \frac{2}{7} \cdot \frac{3}{5} = \frac{2 \cdot 3}{7 \cdot 5} = \frac{6}{35}$$

$$\textcircled{c} \frac{1}{6} \cdot \frac{4}{7} = \frac{1}{\cancel{6}_3} \cdot \frac{\cancel{4}^2}{7} = \frac{1 \cdot 2}{3 \cdot 7} = \frac{2}{21}$$

$$\textcircled{d} \frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \cdot \frac{5}{3} = \frac{2 \cdot 5}{7 \cdot 3} = \frac{10}{21}$$

- Cancellation: As long as  $a \neq 0$  or  $c \neq 0$

$$\frac{ab}{ac} = \frac{b}{c}$$

Ex 2: Can you cancel in the following?

$$\textcircled{a} \frac{3(4+2)}{3(5+7)} = \frac{\cancel{3}(4+2)}{\cancel{3}(5+7)} \quad \text{Yes!}$$

$$= \frac{6}{12} = \frac{1}{2}$$

$$\textcircled{b} \frac{3+4}{3+5}$$

$$= \frac{7}{8}$$

Not in its form.  
Let's check if it simplifies.  
It doesn't so we are done.

$$\textcircled{c} \frac{2x(x+2)}{5x(x-7)} = \frac{2 \cdot \cancel{x} \cdot (x+2)}{5 \cdot \cancel{x} \cdot (x-7)} = \frac{2x+4}{5x-35} \quad \text{Yes!}$$

$$\textcircled{d} \frac{2(x+2)}{5(x+2)} = \frac{2 \cdot \cancel{(x+2)}}{5 \cdot \cancel{(x+2)}} = \frac{2}{5} \quad \text{Yes!}$$

Factoring and cancelling:

Ex 3: Simplifying the rational expressions

$$\textcircled{a} \frac{m-7}{m^2-49} = \frac{\cancel{m-7}}{(\cancel{m-7})(m+7)} \quad \left( \begin{array}{l} \text{by difference of} \\ \text{squares formula} \end{array} \right)$$

$$= \frac{1}{m+7}$$

$$\textcircled{b} \frac{3x^2-5x-2}{x^2-4} \cdot \frac{2x-2}{x-3}$$

$$\textcircled{1} \begin{aligned} 3x^2-5x-2 &= 3x^2+x-6x-2 & 3 \cdot (-2) &= -6 \\ &= x(3x+1)-2(3x+1) & \begin{array}{l} \wedge \\ \boxed{1-6=-5} \\ 2-3=-1 \end{array} \\ &= (x-2)(3x+1) \end{aligned}$$

$$\textcircled{2} x^2-4 = (x-2)(x+2)$$

$$\rightarrow = \frac{\cancel{(x-2)}(3x+1)}{\cancel{(x-2)}(x+2)} \cdot \frac{2(x-1)}{x-3} = \frac{2(3x+1)(x-1)}{(x+2)(x-3)}$$

Expand

	x	-1
3x	3x <sup>2</sup>	-3x
1	x	-1

	x	-3
x	x <sup>2</sup>	-3x
2	2x	-6

$$= \frac{2(3x^2-2x-1)}{x^2-x-6} = \frac{6x^2-4x-2}{x^2-x-6}$$

$$-\frac{2(2x - 2x - 1)}{x^2 - x - 6} = \frac{6x - 1x - 2}{x^2 - x - 6}$$

$$\textcircled{c} \frac{\overset{\textcircled{1}}{4y^2 - 81}}{\underset{\textcircled{2}}{2y^2 - 23y + 63}} \div \frac{\overset{\textcircled{3}}{y^2 - y - 6}}{\underset{\textcircled{4}}{y^2 - 5y - 14}}$$

Let's factor each one of these

$$\textcircled{1} 4y^2 - 81 = (2y)^2 - (9)^2 = (2y - 9)(2y + 9)$$

$$\begin{aligned} \textcircled{2} 2y^2 - 23y + 63 &= 2y^2 - 9y - 14y + 63 \\ &= y(2y - 9) - 7(2y - 9) \\ &= (y - 7)(2y - 9) \end{aligned}$$

$$63 \cdot 2 = 2 \cdot 7 \cdot 9 = 2 \cdot 3^2 \cdot 7$$

$$\begin{array}{r} \wedge \\ -2 - 63 \\ -3 - 14 \cdot 3 \\ -6 - 21 \\ -7 - 18 \end{array}$$

$$\textcircled{-9 - 14 = -23}$$

$$\textcircled{3} y^2 - y - 6 = (y - 3)(y + 2)$$

$$\textcircled{4} y^2 - 5y - 14 = (y - 7)(y + 2)$$

Let's put it all together

$$= \frac{\cancel{(2y - 9)}(2y + 9)}{(y - 7)\cancel{(2y - 9)}} \div \frac{(y - 3)\cancel{(y + 2)}}{(y - 7)\cancel{(y + 2)}} = \frac{2y + 9}{y - 7} \div \frac{y - 3}{y - 7}$$

$$= \frac{2y + 9}{\cancel{y - 7}} \cdot \frac{\cancel{y - 7}}{y - 3} = \frac{2y + 9}{y - 3}$$

## Combining fractions and simplifying

Ex 4: Simplifying the expression

$$\textcircled{a} \frac{\frac{9}{x} + \frac{x}{4}}{\frac{2}{x} - 5} = \frac{\frac{9}{x} + \frac{x}{4}}{\frac{2}{x} - \frac{5}{1}}$$

of denominators  
GCF(x, 4, 1) = 4x

$$= \frac{\left(\frac{9}{x} + \frac{x}{4}\right)(4x)}{\frac{2 \cdot 4x}{x} - \frac{5 \cdot 4x}{1}} = \frac{\frac{9 \cdot 4x}{\cancel{x}} + \frac{x \cdot 4x}{\cancel{4}}}{\frac{8x}{\cancel{x}} - \frac{20x}{1}} = \frac{36 + x^2}{8 - 20}$$

$$= \frac{\left(\frac{9}{x} + \frac{x}{4}\right)(4x)}{\left(\frac{2}{x} - \frac{5}{1}\right)(4x)} = \frac{\frac{\cancel{4}^{\cancel{4}}}{\cancel{x}} + \frac{\cancel{4}}{\cancel{4}}}{\frac{2 \cdot \cancel{4x}}{\cancel{x}} - \frac{5 \cdot \cancel{4x}}{1}} = \frac{36 + x^2}{8 - 20x}$$

(b)  $\frac{\frac{1}{x+3} - 2}{\frac{6}{x+3} + 7} = \frac{\frac{1}{x+3} - \frac{2}{1}}{\frac{6}{x+3} + \frac{7}{1}}$

$\nearrow$  of denominators  
 GCF  $(x+3, 1) = x+3$

$$= \frac{\left(\frac{1}{x+3} - \frac{2}{1}\right)(x+3)}{\left(\frac{6}{x+3} + \frac{7}{1}\right)(x+3)} = \frac{\frac{\cancel{x+3}}{\cancel{x+3}} - \frac{2(x+3)}{1}}{\frac{6\cancel{x+3}}{\cancel{x+3}} + \frac{7(x+3)}{1}} = \frac{1 - 2(x+3)}{6 + 7(x+3)}$$

$$= \frac{1 - 2x - 6}{6 + 7x + 21} = \frac{-2x - 5}{7x + 27}$$