Lesson 3: Simplifying Algebraic Expressions Pt 2

Radical rules: $\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Warning: $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$
i.e. square root does not distribute over addition/subtraction
ex. The expression $\sqrt{x^2+4}$ cannot be further simplified

- The most simplified version of an expression with square roots should not contain
  1. perfect squares
  2. radicals in the denominators → For the purpose of this class, having a radical in the denominator may be okay
     ↓
     multiply by the conjugate
     (conjugate of $a+\sqrt{b}$ is $a-\sqrt{b}$ and vice versa)

Ex 1: Simplify the expression

(a) $\sqrt{1200} = \sqrt{12\cdot 100} = \sqrt{12}\cdot\sqrt{100} = \sqrt{3\cdot 4}\cdot\sqrt{100} = \sqrt{3}\cdot\sqrt{4}\cdot\sqrt{100} = \sqrt{3}\cdot 2\cdot 10 = 20\sqrt{3}$

(b) $\sqrt{\frac{64}{3}} = \frac{\sqrt{64}}{\sqrt{3}} = \frac{8}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{\sqrt{3\cdot 3}} = \frac{8\sqrt{3}}{\sqrt{9}} = \frac{8\sqrt{3}}{3}$

(c) $\frac{4}{x+\sqrt{3}}\cdot\frac{(x-\sqrt{3})}{(x-\sqrt{3})} = \frac{4(x-\sqrt{3})}{x^2-3}$

Side note:
$(a-b)(a+b) = a^2-b^2$
$(x-\sqrt{3})(x+\sqrt{3}) = x^2-(\sqrt{3})^2 = x^2-3$

(d) $\frac{5-x}{\sqrt{x}+\sqrt{5}}\cdot\frac{(\sqrt{x}-\sqrt{5})}{(\sqrt{x}-\sqrt{5})} = \frac{(5-x)(\sqrt{x}-\sqrt{5})}{(\sqrt{x})^2-(\sqrt{5})^2} = \frac{(5-x)(\sqrt{x}-\sqrt{5})}{x-5}$

$$\frac{}{\sqrt{x}+\sqrt{5}} \cdot \frac{}{(\sqrt{x}-\sqrt{5})} = \frac{}{(\sqrt{x})^2-(\sqrt{5})^2} \qquad x-5$$

$$= -\frac{(x-5)(\sqrt{x}-\sqrt{5})}{x-5} = -(\sqrt{x}-\sqrt{5})$$

$$= \sqrt{5}-\sqrt{x}$$

- When $a \geq 0$, $\sqrt{a^2} = a$ (What happens if $a < 0$?)

  ex. $a = -4$ $\sqrt{(-4)^2} = \sqrt{16} = 4$

  In general, For any x positive or negative

  $$\sqrt{x^2} = |x|$$

  (absolute value)

Ex 2: Simplifying the expression (assume x is positive)

(a) $\sqrt{\frac{24x^3}{6x}} = \sqrt{4x^2} = \sqrt{4} \cdot \sqrt{x^2} = 2x$

(b) $\sqrt{\frac{11x}{4x^3}} = \sqrt{\frac{11}{4x^2}} = \frac{\sqrt{11}}{\sqrt{4x^2}} = \frac{\sqrt{11}}{2x}$