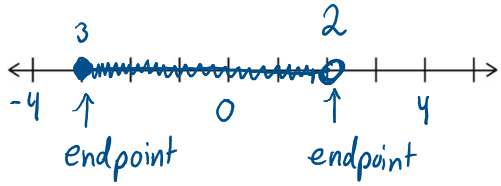
Lesson 5: Domain and Range of a Function

Intervals of Real Numbers

- Different ways to represent sets of numbers
  - ↳ these three represent the same thing

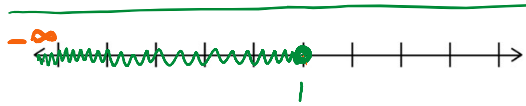
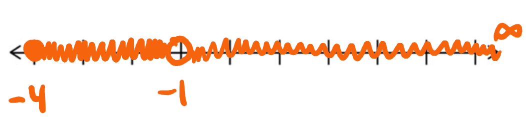
① 

② $-8 \leq x < 3$

③ $[-8, 3)$

- When using interval notation
  - ↳ "[" or "]" means the endpoint is included (solid dot) [in the interval]
  - ↳ "(" or ")" means the endpoint is excluded (hollow dot) [not in the interval]
  - ↳ We always use "(" with $-\infty$ and ")" with $+\infty$
  - ↳ To indicate two seperate intervals, use the union symbol "$\cup$"

Ex 1: Fill in the following chart:

Number line	Inequality	Interval notation
	Ans 1: $-\infty < x \leq 1$ Ans 2: $x \leq 1$	$(-\infty, 1]$
	$0 < x \leq 2$, $5 < x < 9$	$(0, 2] \cup (5, 9)$
	$-4 \leq x < -1$, $-1 < x < \infty$	$[-4, -1) \cup (-1, \infty)$

Domains of Functions

the domain of a function is the set of all possible inputs

Domains of Functions

- Recall that the domain of a function is the set of all possible inputs
- For many functions, we can plug in any real number
  ↳ In this case, the domain $(-\infty, \infty)$
  ex. If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ (which is a polynomial), then the domain is $(-\infty, \infty)$.

- In this class, there are two situations where we must restrict the domain of a function.
  1. $\frac{1}{x}$ Restriction is $x \neq 0$
  2. $\sqrt{x}$ Restriction is $x \geq 0$

     This extends to $\sqrt[4]{x}$, $\sqrt[6]{x}$, $\sqrt[n]{x}$ as long as n is even.

     Special Case: $\sqrt[3]{x}$ so there is no restriction $\Rightarrow$ Domain is $(-\infty, \infty)$
     We know $(-1)^3 = -1$ but $(-1)^2 = 1$

     This extends to $\sqrt[5]{x}$, $\sqrt[7]{x}$, $\sqrt[n]{x}$ as long as n is odd.

- For real-world applications, you may be asked to determine the domain based on what makes sense in the situation.
  ex. Find the distance between Aly and Nimay.
  Distance can never be negative. So domain $x \geq 0$.

Ex 2: Find the domain of the function

(a) $f(x) = 4x^4 + 7x^3 - 10$

B/c this is polynomial $\Rightarrow$ Domain is $(-\infty, \infty)$

(b) $g(x) = \frac{1}{2x-5}$

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Fractions imply denominator $\neq 0$ for domain

$$2x-5 \neq 0$$

Solve for x.

$$\frac{2x}{2} \neq \frac{5}{2}$$

$$x \neq 5/2 \Rightarrow \left(-\infty, \tfrac{5}{2}\right) \cup \left(\tfrac{5}{2}, \infty\right)$$

(c) $h(t) = \dfrac{1}{t^2+1}$

Fractions imply denominator $\neq 0$ for domain.

$$t^2+1 \neq 0$$

Solve for t. $\quad t^2 \neq -1$

$t \neq \pm\sqrt{-1}$ <u>BUT</u> this never happens

$\Rightarrow$ The denominator can <u>never</u> be 0.

$\Rightarrow$ Domain is $(-\infty, \infty)$

(d) $k(x) = \sqrt{8-4x}$

Square roots imply inside ≥ 0 for domain

$$8-4x \geq 0$$

Solve for x. $\quad \dfrac{8}{4} \geq \dfrac{4x}{4}$

$2 \geq x \quad \Rightarrow$ Interval $(-\infty, 2]$

$-\infty < x \leq 2$

For more complex functions, you may have to combine domains of different parts of the function.

<u>Ex 3</u>: Find the domain of the function

(a) $f(x) = 3\sqrt{2x+9}$

ⓐ $f(x) = \sqrt[3]{2x+9}$

Cube root: $\sqrt[n]{x}$ where n is 3 which is odd

Domain of $\sqrt[n]{x}$ is $(-\infty, \infty)$

So Domain of f is $(-\infty, \infty)$

ⓑ $R(y) = \dfrac{3y+5}{\sqrt{2y-6}}$

Restrictions: We have ① $\sqrt{2y-6}$ and ② $\dfrac{1}{\sqrt{2y-6}}$

① $\sqrt{2y-6} \Rightarrow 2y-6 \geq 0$

$2y \geq 6$

$y \geq 3$

② $\dfrac{1}{\sqrt{2y-6}} \Rightarrow \sqrt{2y-6} \neq 0$

$2y-6 \neq 0^2 = 0$

$2y \neq 6$

$y \neq 3$

So domain is $y \geq 3$ and $y \neq 3 \Rightarrow y > 3$ Interval Notation: $(3, \infty)$

ⓒ $g(x) = \dfrac{\sqrt{x-5}}{x-1}$

Restrictions: We have ① $\sqrt{x-5}$ and ② $\dfrac{1}{x-1}$

① $\sqrt{x-5} \Rightarrow x-5 \geq 0$

$x \geq 5$

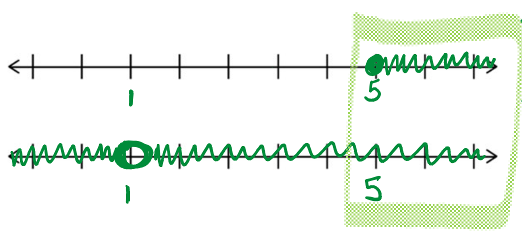
② $\dfrac{1}{x-1} \Rightarrow x-1 \neq 0$

$x \neq 1$

So domain is $x \geq 5$ and $x \neq 1$

$x \geq 5$

$x \neq 1$



Find the common shading

Domain: $[5, \infty)$

Domain and Range from a Graph

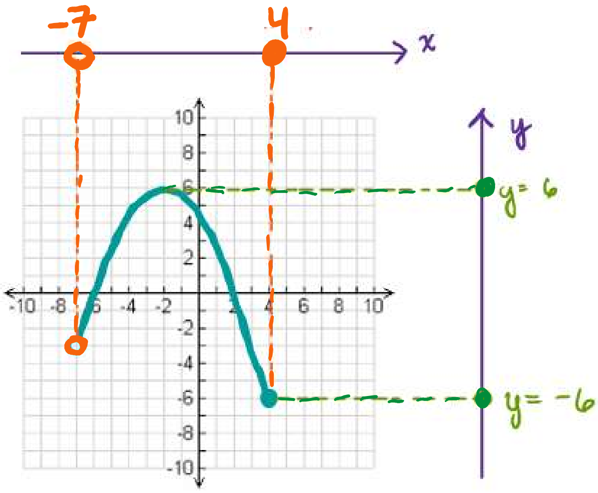
- Recall that the range of a function is the set of all possible outputs ... the domain is all possible x-values, and the range

• Recall that the range of a function is the set of all possible outputs

↳ So for a graph, the domain is all possible x-values, and the range is all possible y-values.

Ex 4: Find the domain and range of the following graphs

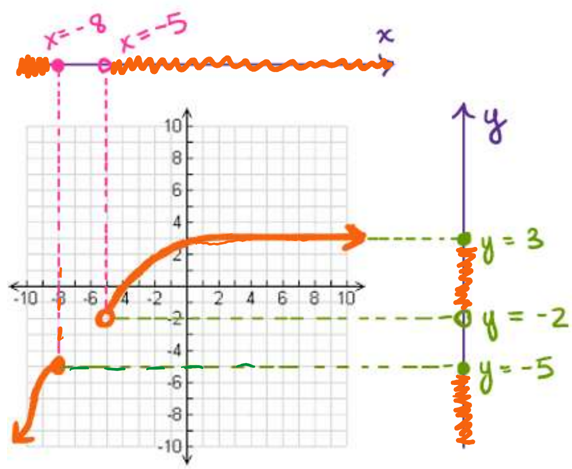
(a)



Domain: $(-7, 4]$

Range: $[-6, 6]$

(b)



Domain: $(-\infty, -8] \cup (-5, \infty)$

Range: $(-\infty, -5] \cup (-2, 3]$