Lesson 6: Function Arithemic & Piecewise Functions

Function Arithmetic

- So far, we have added, subtracted, multiplied, and divided numbers
  ↳ do the same with functions

- With function notation

$$\overbrace{\underbrace{f}_{\text{function}}(\underbrace{x}_{\text{input}})}^{\text{output ("f of x")}} \qquad\qquad \underbrace{(f+g)}_{\text{Function}}(x)$$

- How to define a function? We must say what the function does to an input.

- For f, g are fuctions, we define four potential new functions:

① $f+g$: $(f+g)(x) \stackrel{\text{def}}{=} f(x)+g(x)$

② $f-g$: $(f-g)(x) \stackrel{\text{def}}{=} f(x)-g(x)$

③ $f\cdot g$: $(f\cdot g)(x) \stackrel{\text{def}}{=} (f(x))\cdot(g(x))$

④ $\frac{f}{g}$: $\left(\frac{f}{g}\right)(x) \stackrel{\text{def}}{=} \frac{f(x)}{g(x)}$

☆ For x to be in the domain of $f+g$, $f-g$, or $f\cdot g$, it must be in the domain of f <u>and</u> g

☆ For x to be in the domain of $\frac{f}{g}$, it must be in the domain of f <u>and</u> g, and $g(x) \neq 0$.

$\boxed{\text{In general, you should find the domain Before Simplifying}}$

<u>Ex 1</u>: Given $f(x) = x+2$ and $g(x) = 6-2x$, find:

(a) $(f-g)(4) \stackrel{\text{def}}{=} f(4)-g(4)$

Ex 1: Given

ⓐ $(f-g)(4) \underline{\text{def}}\ f(4) - g(4)$

$f(4) = 4+2 = 6$

$g(4) = 6-2(4) = 6-8 = -2$

$= 6 + (+2)$

$= 8$

ⓑ $\left(\frac{f}{g}\right)(5) \underline{\underline{\text{def}}}\ \frac{f(5)}{g(5)}$

$f(5) = 5+2 = 7$

$g(5) = 6-2(5) = 6-10 = -4$

$= \frac{7}{-4} = -\frac{7}{4}$

Ex 1: Given $f(x) = x+2$ and $g(x) = 6-2x$, find:

ⓒ $\left(\frac{f}{g}\right)(3) \underline{\underline{\text{def}}}\ \frac{f(3)}{g(3)}$

$f(3) = 3+2 = 5$

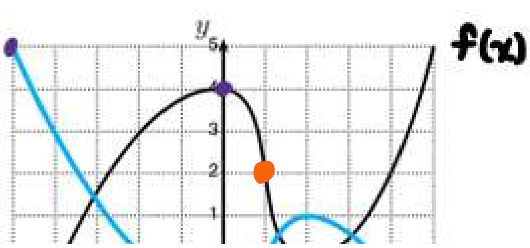
$g(3) = 6-2(3) = 6-6 = 0$

$= \frac{5}{0}$ No-no

= undefined

ⓓ $\left(\frac{f}{g}\right)(x) \underline{\underline{\text{def}}}\ \frac{f(x)}{g(x)} = \frac{x+2}{6-2x}$

Ex 2: Evaluate the following:

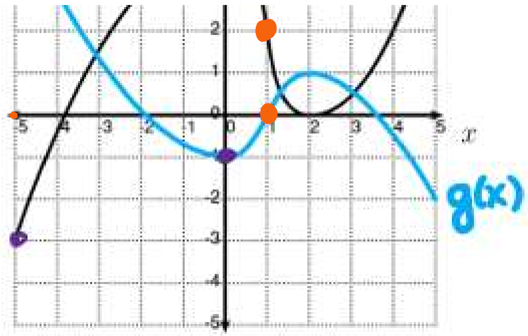


ⓐ $(g+f)(-5) \underline{\underline{\text{def}}}\ g(-5) + f(-5)$

$= 5 + (-3) = 2$

(by the graph)

ⓑ $\left(\frac{f}{g}\right)(0) \underline{\underline{\text{def}}}\ \frac{f(0)}{g(0)} = \frac{4}{-1} = -4$



ⓑ $\left(\frac{f}{g}\right)(0) \stackrel{def}{=} \frac{f(0)}{g(0)} = \frac{4}{-1} = -4$

(by the graph)

ⓒ $\left(\frac{g}{f}\right)(1) \stackrel{def}{=} \frac{g(1)}{f(1)} = \frac{0}{2} = 0$

(by the graph)

Ex 3: Given $f(x) = 3x^2 + 2x - 5$ and $g(x) = 3x + 5$, find

ⓐ $(f+g)(x) \stackrel{def}{=} f(x) + g(x)$

$= 3x^2 + 2x - \cancel{5} + 3x + \cancel{5}$

$= 3x^2 + 5x$

ⓑ $(f-g)(x) \stackrel{def}{=} f(x) - g(x)$

$= 3x^2 + 2x - 5 - (3x + 5)$

$= 3x^2 + 2x - 5 - 3x - 5$

$= 3x^2 - x - 10$

ⓒ $(f \cdot g)(x) \stackrel{def}{=} (f(x)) \cdot (g(x))$

$= (3x^2 + 2x - 5)(3x + 5)$

$= 9x^3 + 6x^2 + 15x^2 - 15x + 10x - 25$

$= 9x^3 + 21x^2 - 5x - 25$

	$3x^2$	$2x$	-5
$3x$	$9x^3$	$6x^2$	$-15x$
5	$15x^2$	$10x$	-25

Ex 3: Given $f(x) = 3x^2 + 2x - 5$ and $g(x) = 3x + 5$, find

ⓓ $\left(\frac{f}{g}\right)(x) \stackrel{def}{=} \frac{f(x)}{g(x)} = \frac{3x^2 + 2x - 5}{3x + 5}$

Let's factor $3x^2 + 2x - 5$

$= 3x^2 - 3x + 5x - 5$

$= 3x(x-1) + 5(x-1)$

$= (3x+5)(x-1)$

-15

$-1 + 15 = 14$

$\boxed{-3 + 5 = 2}$

$$= (3x+5)(x-1)$$

$$= \frac{\cancel{(3x+5)}(x-1)}{\cancel{3x+5}} = x-1$$

ⓔ The domain of $\frac{f}{g}$ is domain of f, domain of g, and when $g(x) \neq 0$.

f is polynomial → Domain $(-\infty, \infty)$

g is polynomial → Domain $(-\infty, \infty)$

$3x+5 \neq 0$
$3x \neq -5$
$x \neq -5/3$

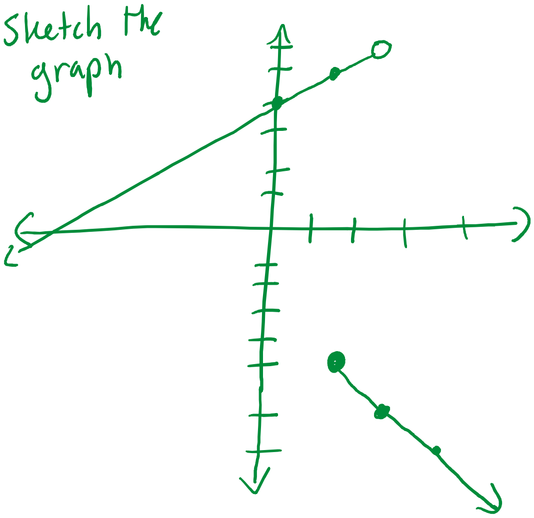
is $\left(-\infty, -\frac{5}{3}\right) \cup \left(-\frac{5}{3}, \infty\right)$

Piecewise Functions

- Sometimes, different formulas are needed for different parts of the domain of a function

Ex 4: Let $f(x) = \begin{cases} x+4, & x < 2 \\ -3x+1, & x \geq 2 \end{cases}$

ⓐ Sketch the graph



I know

① $x+4$, $-3x+1$ are lines

② For $x+4$ @ $x=2$, I have a hole.
When $x=2$, $y=2+4=6$
Hole @ $(2,6)$

③ For $-3x+1$ @ $x=2$, I have a filled dot
When $x=2$, $y=-3(2)+1=-5$
Fill dot @ $(2,-5)$

④ When $y=1x+4$
Slope $m = 1 = \frac{1}{1}$

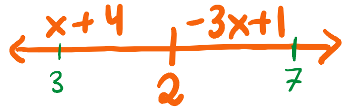
⑤ When $y=-3x+1$
Slope $m=-3 = -\frac{3}{1}$

ⓑ Find $f(-3)$, $f(2)$, and $f(7)$

$f(-3) = (-3)+4 = 1$

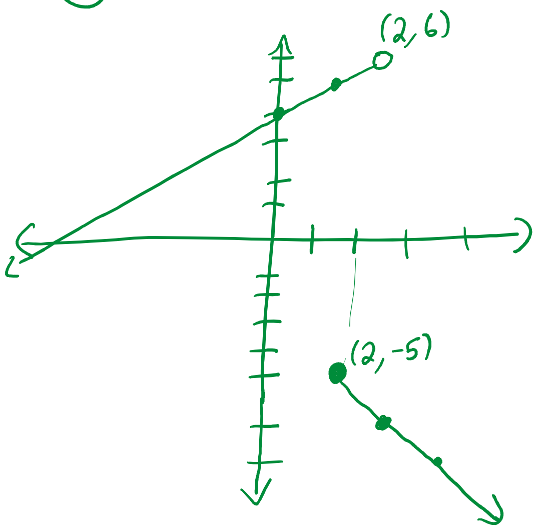
$f(2) = -3(2)+1 = -6+1 = -5$

$f(7) = -3(7)+1 = -21+1 = -20$



[b/c where is the equal sign]

(c) Find the domain and the range of $f(x)$.



<u>Domain:</u> $(-\infty, 2) \cup [2, \infty) = (-\infty, \infty)$

<u>Range:</u> 6 is a stopping point and will have paranthesis

$\Rightarrow (-\infty, 6)$

<u>Ex 5:</u> A car towing company charges a flat rate of \$80 to tow a vehicle to a location within 10 miles, and charges \$4 per mile for every additional mile over 10.

Write a piecewise function $C(m)$ describing the cost of towing as a function of miles.

m is miles.

$$0 \le m \le 10 \Rightarrow C(m) = \$80$$

$$m > 10 \Rightarrow C(m) = \underbrace{\$80}_{\text{for the 10 miles}} + \underbrace{\$4(m-10)}_{\text{Every mile after 10 miles}}$$

<u>So:</u> $C(m) = \begin{cases} 80, & 0 \le m \le 10 \\ 80 + 4(m-10), & m > 10 \end{cases}$