Reminders

Exam 1 Tonight @ 8pm-9pm in ES2107

15 questions
Multiple Choice
TI 30Xa only

Written HW 3 Due Friday

Lesson 8: Graph Transformations

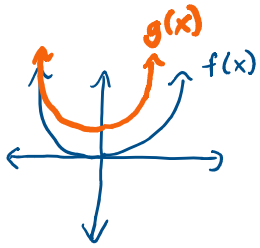
Types of Transformations:

(1) Vertical Shift/Translation { shift up / shift down

$k > 0$: shift up
$k < 0$: shift down

↳ $g(x) = f(x) + k$

ex. If $k = 1$



(2) Horizontal Shift/Translation { shift left / shift right

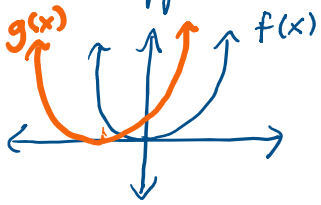
$h > 0$: left shift
$h < 0$: right shift

↳ $g(x) = f(x+h)$

☆ h positive/negative has opposite effect as in vertical shift

ex. If $h = 2$

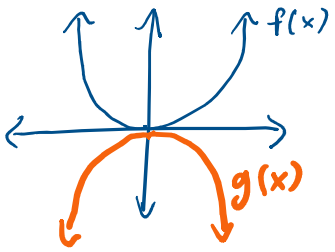
b/c $h > 0$



(3) Vertical Reflection (i.e. reflection over the x-axis)

↳ $g(x) = -f(x) = (-1)f(x)$

ex.

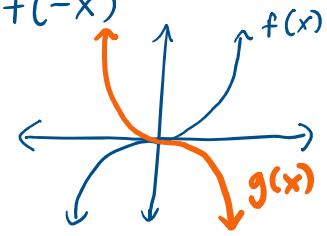


(4) Horizontal Reflection (i.e. reflection over the y-axis)

④ Horizontal Reflection (i.e. reflection over the y-axis)

↳ $g(x) = f(-x)$

ex.



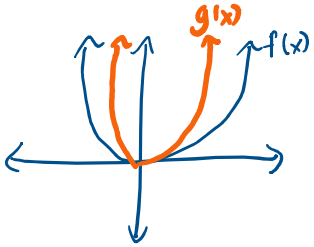
⑤ Vertical Scaling { stretch / compression

$a > 1$: stretch

$0 < a < 1$: compression/squish

↳ $g(x) = af(x)$ where a is positive

ex.



$a = \frac{1}{2}$

⑥ Horizontal Scaling { stretch / compression

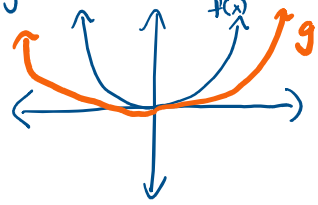
$0 < b < 1$: strech

$b > 1$ compression

↳ $g(x) = f(bx)$

↳ b greater/less than 1 has opposite effect as in vertical scaling

ex.



$b = \frac{1}{2}$

Remarks: ① Reflection and scaling can happen together.

ex. $g(x) = -2f(x)$

$= (-1)[2f(x)]$

↓ Reflection ↘ Stretch by a factor 2

② Vertical whatevers (scaling/compressions/stretchs) change the outside of the function

③ Horizontal whatevers changes the inside.

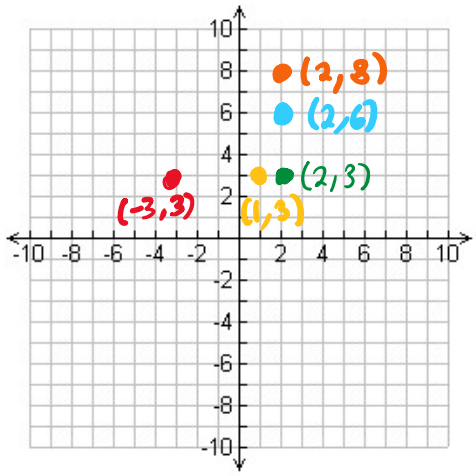
We are going to work with

$$y = a\,f(bx+h) + k$$

(horizontal: $bx+h$; vertical: a and k)

Ex 1: Suppose the point $(2,3)$ is contained in the graph of f. $\rightarrow f(2)=3$

Find a point that must be in the graph of g. $\rightarrow (x, g(x))$



ⓐ $g(x) = f(x) + 5$

$g(2) = f(2) + 5 = 3 + 5 = 8$ } $(2, 8)$

ⓑ $g(x) = f(x+5)$

$f(2) = 3$

$x+5 = 2$

$x = -3$

$g(-3) = 3 \Rightarrow (-3, 3)$

ⓒ $g(x) = 2\,f(x)$

$g(2) = 2\,f(2) = 2 \cdot 3 = 6$ } $(2, 6)$

ⓓ $g(x) = f(2x)$

$f(2) = 3$

$2x = 2$

$x = 1$

$g(1) = 3$

Ex 2: Let $f(x) = |x|$

① Write $g(x)$ in terms of $f(x)$. ✓

② Explain how you would transform the graph of f to draw the graph of g.

③ Sketch the graph of g.

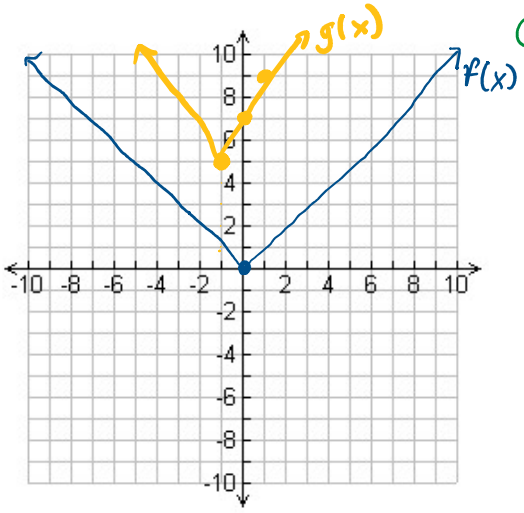
ⓐ $g(x) = 2|x+1| + 5$

$= 2\,f(x+1) + 5$

Inside $\Rightarrow$ Horizontal $\Rightarrow x+1$

Shift left by 1

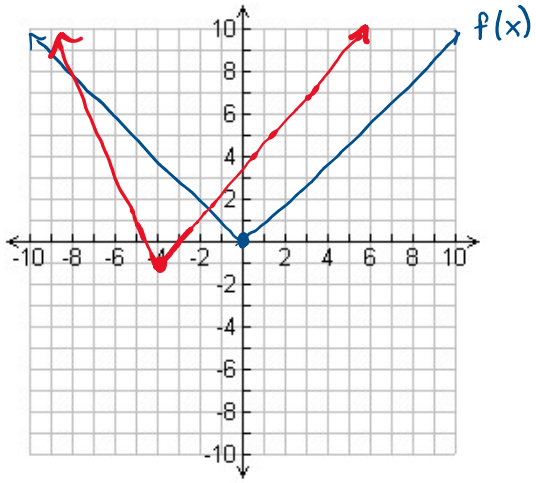
Shift Left by 1



Outside ⇒ Vertical ⇒ $\cdots + 5$
Vertical Shift up by 5
⇒ $2 \cdots$
Vertical Stretch by 2

(b) $g(x) = |2x+4| - 1$
$= f(2x+4) - 1$

Vertical Shift by -1 ⇒ Down by 1
Horizontal Shift by 4 ⇒ Left by 4
Horizontal Compression by 2 ⇒ Shrinking by 2



Ex 3: For each function and transformation described, write a formula for the transformed function

(a) $f(x) = x^2$
① Shift right by 3 ⇒ $f(x-3)$
② Reflection over the x-axis ⇒ $-f(x)$
} $g(x) = -f(x-3)$
$= -(x-3)^2$

(b) $f(x) = \sqrt{x}$
① Stretch vertically by a factor of 5 ⇒ $5f(x)$
② Shift up by 2 ⇒ $f(x) + 2$
} $g(x) = 5f(x) + 2$
$= 5\sqrt{x} + 2$

② Shift up by 2 $\Rightarrow f(x)+2$ $= 5\sqrt{x-1}+2$

(c) $f(x)=\frac{1}{x}$

① Shift left by 3 $\Rightarrow f(x+3)$

② Shift up by 4 $\Rightarrow f(x)+4$

$g(x) = f(x+3)+4$

$= \frac{1}{x+3}+4$