1. You should be able to determine the difference quotient given a function.
    a. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for
    $$f(x) = 5x^2 - 3x + 7$$

    b. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for
    $$f(x) = -4x^2 + 12x - 1$$

    c. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for
    $$f(x) = 2x^3 - x^2 + 6$$

    d. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for
    $$f(x) = x^2 + 6x + 70$$

2. You should be able to transform a given point given a transformation.
    a. The graph of $y = f(x)$ contains the point $(4, -3)$. What is the corresponding point on the graph of $y = f(x + 5)$?

    b. The graph of $y = f(x)$ contains the point $(-2, 6)$. What is the corresponding point on the graph of $y = f(x) + 4$?

    c. The graph of $y = f(x)$ contains the point $(3, -2)$. What is the corresponding point on the graph of $y = 3 \cdot f(x)$?

    d. The graph of $y = f(x)$ contains the point $(8, -1)$. What is the corresponding point on the graph of $y = f(-x)$?

    e. The graph of $y = f(x)$ contains the point $(-2, -6)$. What is the corresponding point on the graph of $y = f(x - 11)$?

    f. The graph of $y = f(x)$ contains the point $(3, 17)$. What is the corresponding point on the graph of $y = f(x) - 5$?

3. You should be able to compute the discriminant of a quadratic function and determine the type and number of solutions to the equation.
   a. Determine the discriminant of the polynomial function $f(x) = x^2 - 5x + 6$ and the number of real solutions to the equation $f(x) = 0$.

   b. Determine the discriminant of the polynomial function $f(x) = 4x^2 - 12x + 9$ and the number of real solutions to the equation $f(x) = 0$.

   c. Determine the discriminant of the polynomial function $f(x) = 2x^2 + 4x + 7$ and the number of real solutions to the equation $f(x) = 0$.

   d. Determine the discriminant of the polynomial function $f(x) = 3x^2 + x - 4$ and the number of real solutions to the equation $f(x) = 0$.

4. You should be able to solve a quadratic equation.
   a. Solve the equation $x^2 - 12x + 36 = 0$.

   b. Solve the equation $x^2 + 10x + 25 = 0$

   c. Solve the equation $4x^2 - 20x + 25 = 0$

   d. Solve the equation $9x^2 + 30x + 25 = 0$

   e. Solve the equation $x^2 - 12x + 36 = 0$

   f. Solve the equation $16x^2 - 24x + 9 = 0$

5. You should be able to write a quadratic function into vertex form.
   a. Rewrite the given quadratic function in standard form $f(x) = a(x - h)^2 + k$.
   $$f(x) = 2x^2 + 8x + 6$$

   b. Rewrite the given quadratic function in standard form $f(x) = a(x - h)^2 + k$.
   $$f(x) = x^2 - 6x + 4$$

   c. Rewrite the given quadratic function in standard form $f(x) = a(x - h)^2 + k$.
   $$f(x) = -3x^2 + 12x - 7$$

   d. Rewrite the given quadratic function in standard form $f(x) = a(x - h)^2 + k$.
   $$f(x) = 4x^2 - 4x + 9$$

6. You should be able to answer problems that are applications (i.e. word problems) of quadratic functions.

   a. A rectangular field is to be fenced off, and then divided into three sections by fencing running parallel to one of the sides. If 640 meters of fencing is used, find the maximum total area that can be enclosed.
   Solution: Max area = 12, 800 (dimensions 160 m x 80 m)

   b. A small business sells handmade notebooks. When the price is set at $20, they sell 200 notebooks per month. For every $2 increase in price, they sell 10 fewer notebooks. Write a revenue function in terms of the number of $2 price increases. What is the maximum revenue?
   Solution: Revenue function $R(n) = (20 + 2n)(200 - 10n)$ and max revenue is $4,500

   c. A ball is thrown upward from the top of a 64-foot building with an initial velocity of 32 ft/s. Its height (in feet) after t seconds is given by
   $$h(t) = -16t^2 + 32t + 64$$
   At what time does the ball reach its maximum height?
   Solution: $t = 1$ seconds

   d. A farmer has 300 meters of fencing to enclose a rectangular pasture along a straight river. No fencing is needed along the river. Let x represent the width perpendicular to the river. Write a function for the area in terms of x. What is the maximum area?
   Solution: A(x)=x(300-2x) and max area is 11,250 m^2

   e. A company manufactures open-top boxes by cutting squares of side length x from each corner of a 20-inch by 14-inch sheet of metal and folding up the sides. Write a function for the volume of the box in terms of x. What is the maximum volume?
   Solution: Volume = 274 in^3

7. You should be able to determine the y-intercept(s) of a function.

    a. Determine the (x, y) coordinates of the y-intercept(s) of the graph of
$$f(x) = x^2 - 49$$

    b. Determine the (x, y) coordinates of the y-intercept(s) of the graph of
$$f(x) = x^2 - 12x + 36$$

    c. Determine the (x, y) coordinates of the y-intercept(s) of the graph of
$$f(x) = 3x^2 + 5x - 12$$

    d. Determine the (x, y) coordinates of the y-intercept(s) of the graph of
$$f(x) = -2x^2 + 8x + 15$$

    e. Find the y-intercept(s) of the function
$$f(x) = 5x(x - 4)(x + 1)(x - 7)$$

    f. Find the y-intercept(s) of the function
$$f(x) = -3x(x + 5)(x - 2)(x + 8)$$

    g. Find the y-intercept(s) of the function
$$f(x) = 12x(x - 6)(x + 3)$$

    h. Find the y-intercept(s) of the function
$$f(x) = -2x(x + 4)(x - 1)(x - 9)$$

8. You should be able to determine the zeros of a function.

    a. If $x - 2$ is a factor of the polynomial $p(x) = x^3 - 3x^2 - 4x + 12$, find the zeros of $p(x)$.

    b. If $x + 1$ is a factor of the polynomial $p(x) = x^3 + 2x^2 - 5x - 6$, find the zeros of $p(x)$.

    c. If $x - 3$ is a factor of the polynomial $p(x) = x^3 - x^2 - 12x + 12$, find the zeros of $p(x)$.

    d. If $x + 4$ is a factor of the polynomial $p(x) = x^3 + 5x^2 - 8x - 40$, find the zeros of $p(x)$.

9. You should be able to determine the vertical and horizontal asymptotes of a rational function.

a. Determine the vertical and horizontal, if possible, asymptote(s) of

$$f(x) = \frac{2x}{x-3}$$

b. Determine the vertical and horizontal, if possible, asymptote(s) of

$$f(x) = \frac{3x^2 - 4}{x^2 + 1}$$

c. Determine the vertical and horizontal, if possible, asymptote(s) of

$$f(x) = \frac{x^2 - 9}{x^2 - 4}$$

d. Determine the vertical and horizontal, if possible, asymptote(s) of

$$f(x) = \frac{5x^3 + 2x}{x^2 - 1}$$

e. Find the horizontal asymptote(s) of the function

$$f(x) = \frac{4x^2 + 7x - 3}{2x^2 - 5}$$

f. Find the horizontal asymptote(s) of the function

$$f(x) = \frac{6x - 1}{x^2 + 4}$$

g. Find the horizontal asymptote(s) of the function

$$f(x) = \frac{7x^3 - 2x}{3x^3 + 9}$$

h. Find the vertical asymptote(s) of the function

$$f(x) = \frac{x + 1}{x^2 + 7x + 10}$$

i. Find the vertical asymptote(s) of the function

$$f(x) = \frac{x + 3}{x^2 + 8x + 15}$$

j. Find the vertical asymptote(s) of the function

$$f(x) = \frac{x^2 - 9}{x^2 + 2x - 15}$$

k. Find the vertical asymptote(s) of the function

$$f(x) = \frac{x^3 - 3x^2 - x + 3}{3x^2 - 12x + 9}$$

10. You should be able to determine the end behavior of a function.

a. Determine the end behavior of $f(x) = 3x(x - 4)^2(x + 1)$.

b. Determine the end behavior of $f(x) = -2x^2(x + 5)(x - 3)^2$.

c. Determine the end behavior of $f(x) = 5(x - 2)^3(x + 6)$.

d. Determine the end behavior of $f(x) = -x(x - 1)^2(x + 4)^3$

11. You should be able to determine the rational function given information about its zeros, horizontal, vertical, and hole.

a. Problem 1:

Which function satisfies:

i) zeros at $x = -6$ and $x = 2$

ii) horizontal asymptote $y = -1$

iii) vertical asymptote at $x = 4$

iv) hole at $x = -3$

A.

$$f(x) = \frac{-1(x+6)(x-2)(x+3)}{(x-4)(x+3)}$$

B.

$$f(x) = \frac{-1(x+6)(x-2)}{(x-4)(x+3)}$$

C.

$$f(x) = \frac{-1(x+6)(x-2)(x+3)}{(x-4)^2(x+3)}$$

D.

$$f(x) = \frac{(x+6)(x-2)(x+3)}{(x-4)(x+3)}$$

E. None of these.

b. Problem 2:

Which function has:

i) zero at $x = 3$

ii) horizontal asymptote $y = 4$

iii) vertical asymptote at $x = -5$

iv) hole at $x = 1$

A.

$$f(x) = \frac{4(x-3)(x-1)}{(x+5)(x-1)}$$

B.

$$f(x) = \frac{4(x-3)}{(x+5)(x-1)}$$

C.

$$f(x) = \frac{4(x-3)(x-1)}{(x+5)^2(x-1)}$$

D.

$$f(x) = \frac{(x-3)(x-1)}{(x+5)(x-1)}$$

E. The function cannot be determined.

c. Problem 3:

Which function satisfies:

i) zeros at $x = -4$ and $x = 1$
ii) horizontal asymptote $y = 1$
iii) vertical asymptote at $x = 6$
iv) hole at $x = -2$

A.
$$f(x) = \frac{(x+4)(x-1)(x+2)}{(x-6)(x+2)}$$

B.
$$f(x) = \frac{(x+4)(x-1)}{(x-6)(x+2)}$$

C.
$$f(x) = \frac{(x+4)(x-1)(x+2)}{(x-6)^2(x+2)}$$

D.
$$f(x) = \frac{2(x+4)(x-1)(x+2)}{(x-6)(x+2)}$$

E. None of these.

d. Problem 4:

Which function has:

i) zero at $x = 7$

ii) horizontal asymptote $y = -2$

iii) vertical asymptote at $x = -3$

iv) hole at $x = 5$

A.

$$f(x) = \frac{-2(x-7)(x-5)}{(x+3)(x-5)}$$

B.

$$f(x) = \frac{-2(x-7)}{(x+3)(x-5)}$$

C.

$$f(x) = \frac{-2(x-7)(x-5)}{(x+3)^2(x-5)}$$

D.

$$f(x) = \frac{2(x-7)(x-5)}{(x+3)(x-5)}$$

E. None of these.

↓

e. Problem 5:

Which of the given functions is a rational function with the following properties?

i) zeros at $x = 2$ and $x = -5$

ii) horizontal asymptote at $y = 3$

iii) vertical asymptote at $x = 4$

iv) hole at $x = -1$

A.

$$f(x) = \frac{3(x-2)(x+5)(x+1)}{(x-4)(x+1)}$$
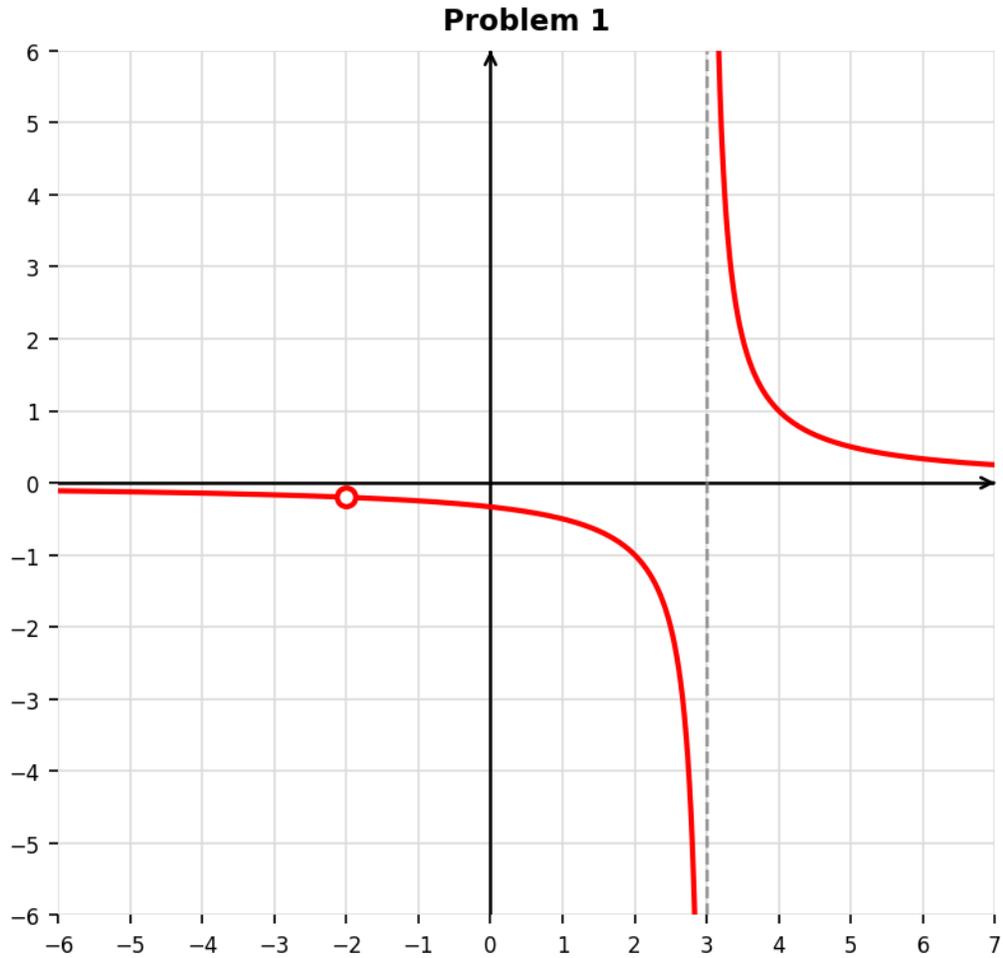
B.

$$f(x) = \frac{3(x-2)(x+5)}{(x-4)(x+1)}$$

C.

$$f(x) = \frac{3(x-2)(x+5)(x+1)}{(x-4)^2(x+1)}$$

D.

$$f(x) = \frac{(x-2)(x+5)(x+1)}{(x-4)(x+1)}$$

E. The function cannot be determined.

12. You should be able to determine the rational function given a graph.
   a. Find an equation for the rational function shown graphed below:

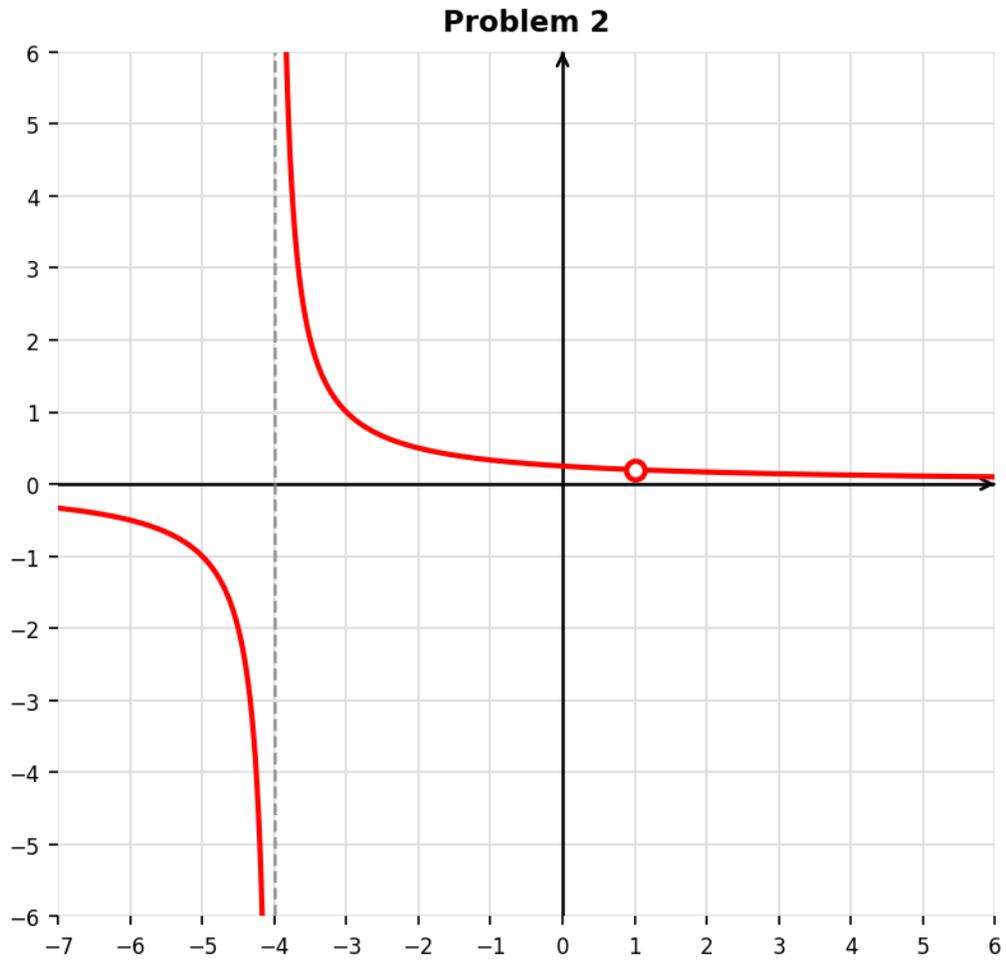**Problem 1**

A) $r(x) = \dfrac{x+2}{(x+2)(x+3)}$

B) $r(x) = \dfrac{x-2}{x+3}$

C) $r(x) = \dfrac{x-2}{(x-2)(x-3)}$

D) $r(x) = \dfrac{x+2}{(x+2)(x+3)}$

E) $r(x) = \dfrac{1}{x-3}$

b. Find an equation for the rational function shown graphed below:

Problem 2
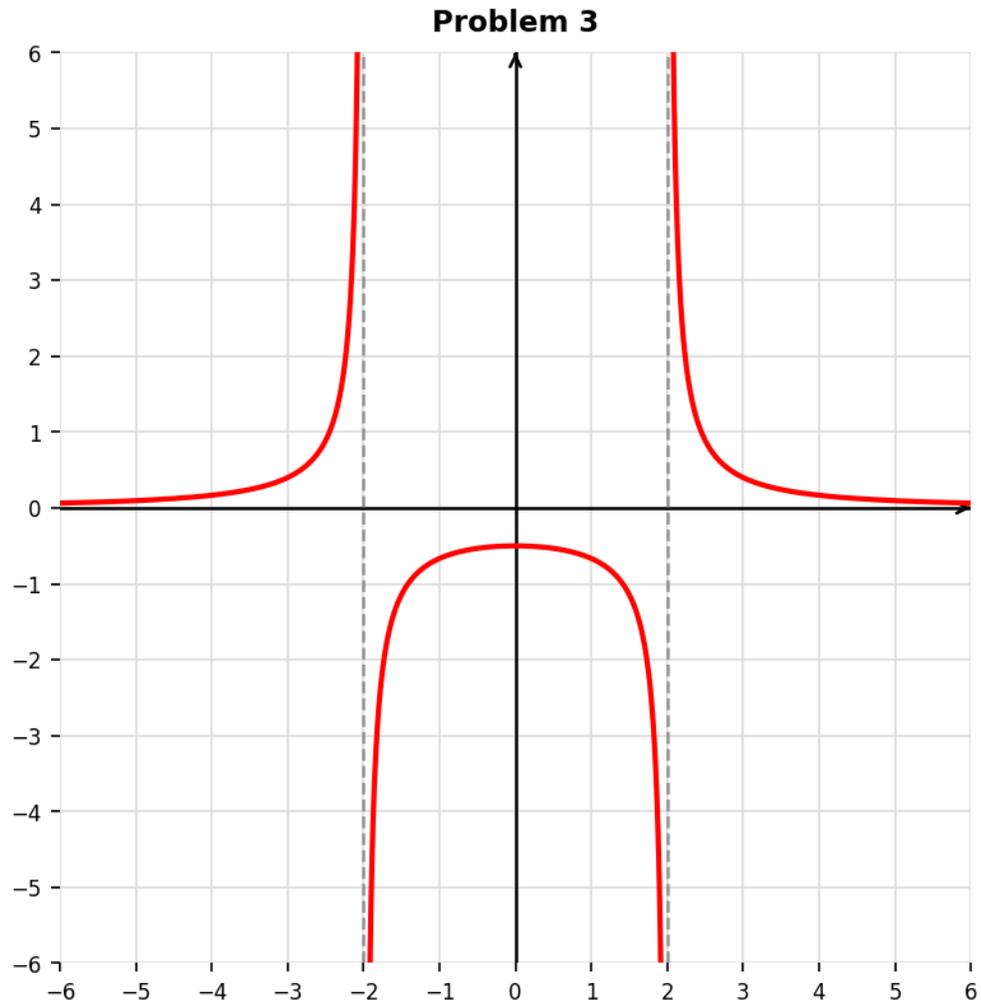

A) $r(x) = \dfrac{1}{x+4}$

B) $r(x) = \dfrac{x-1}{(x-1)(x-4)}$

C) $r(x) = \dfrac{x+1}{(x+1)(x+4)}$

D) $r(x) = \dfrac{x-1}{(x+1)(x+4)}$

E) $r(x) = \dfrac{x-1}{(x-1)(x+4)}$

c. Find an equation for the rational function shown graphed below:

Problem 3

A) $r(x) = \dfrac{2}{x^2+4}$

B) $r(x) = \dfrac{x}{x^2-4}$

C) $r(x) = \dfrac{2}{x^2-4}$

D) $r(x) = \dfrac{2}{(x-2)(x-2)}$

E) $r(x) = \dfrac{x-2}{x^2-4}$