Question 9 of 54

For

$$f(x) = \begin{cases} x^2 - 3, & x \le -2 \\ -4x + 1, & x > -2 \end{cases}$$

Choose the correct statement(s) below.

I. $\lim_{x \to -2} f(x)$ does not exist.

II. $f(x)$ is continuous at $x = -2$.

III. $\lim_{x \to -2^-} f(x) = 1$

- ○ Only II is true
- ○ Only III is true
- ○ Only I and II are true
- ○ Only I and III are true
- ○ Only II and III are true
- ○ Only I is true

$x^2 - 3$ $-4x+1$

$\longleftarrow\!\!\!+\!\!\!\longrightarrow$ -2

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x) = f(-2)$

continuous

$\lim_{x \to -2^-} (x^2 - 3) = (-2)^2 - 3 = 4 - 3 = 1$ $\neq$ DNE

$\lim_{x \to -2^+} (-4x+1) = (-4)(-2) + 1 = 8 + 1 = 9$

I ✓   ~~II~~   III ✓ $\boxed{D}$

Question 14 of 54

Find $f'(2\pi)$ if $f(x) = (x^2 + 4)(x + \sin x)$.

- ○ $16\pi^2 + 4$
- ○ 8π
- ○ $16\pi^2 + 8$
- ○ $12\pi^2 + 2\pi + 4$
- ○ $8\pi^3 + 8\pi$
- ○ -16π

$f(x) = (x^2 + 4)(x + \sin x)$

Product Rule,

$u = x^2 + 4$ $v = x + \sin x$

$u' = 2x$ $v' = 1 + \cos x$

By product rule,

$f' = u'v + uv'$

$= 2x(x + \sin x) + (x^2 + 4)(1 + \cos x)$

$f'(2\pi) = 2(2\pi)(2\pi + \cancelto{0}{\sin(2\pi)})$

$+ ((2\pi)^2 + 4)(1 + \cancelto{1}{\cos(2\pi)})$

$= 4\pi(2\pi) + (4\pi^2 + 4)(2)$

$= 8\pi^2 + 8\pi^2 + 8$

$= 16\pi^2 + 8$ C

Question 18 of 54

The derivative of a function $f(x)$ is found by computing

$$f'(x) = \lim_{h\to 0} \frac{\sqrt[3]{(x+h)^2-1} - \sqrt[3]{x^2-1}}{h}.$$

Which of the following could be $f(x)$?

- $f(x) = \sqrt[3]{(x+h)^2-1}$
- $f(x) = \sqrt[3]{x^2}$
- $f(x) = \dfrac{1}{\sqrt[3]{(x+h)^2-1}}$
- $f(x) = \sqrt[3]{(x+h)^2}$
- $f(x) = \dfrac{1}{\sqrt[3]{x^2-1}}$
- $f(x) = \sqrt[3]{x^2-1}$

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h\to 0} \dfrac{\sqrt[3]{(x+h)^2-1} - \sqrt[3]{x^2-1}}{h}$

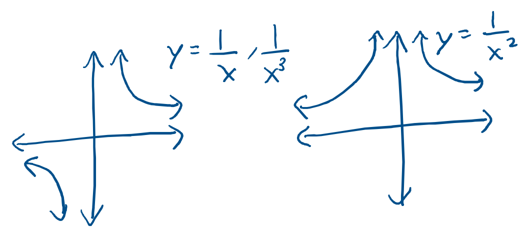
Let's guess $f(x) = \sqrt[3]{x^2-1}$ (F)

Check $f(x+h) \stackrel{?}{=} \sqrt[3]{(x+h)^2-1}$ ✓

Question 19 of 54

Which of the following is **NOT** equal to ∞?

- $\lim_{x\to 2^+} \dfrac{1}{x-2}$
- $\lim_{x\to 4^-} \dfrac{1}{(4-x)^2}$
- $\lim_{x\to 8^-} \dfrac{2}{(x-8)^3}$
- $\lim_{x\to 4^+} \dfrac{1}{(4-x)^2}$
- $\lim_{x\to 8^+} \dfrac{2}{(x-8)^3}$
- $\lim_{x\to 3^+} \dfrac{1}{\sqrt{x-3}}$



~~A~~ $\lim_{x\to 2^+} \dfrac{1}{x-2} = \infty$

~~B~~ $\lim_{x\to 4^-} \dfrac{1}{(4-x)^2} = \lim_{x\to 4^-} \dfrac{1}{(x-4)^2} = \infty$

$\boxed{C}$ $\lim_{x\to 8^-} \dfrac{2}{(x-8)^3} = -\infty$

Question 20 of 54

Find the x value at which $g(x) = xe^x$ has a horizontal tangent line.

- $\frac{1}{2}$
- -2
- 2
- -1
- 0
- 1

$g'(x) = 0$

$g(x) = xe^x$

$u = x \qquad v = e^x$

$u' = 1 \qquad v' = e^x$

$g'(x) = e^x + xe^x = 0$

$e^x(1+x) = 0$

$e^x = 0 \qquad 1 + x = 0$

NEVER $\qquad -1 = x$

Question 22 of 54

Which of the following functions has a hole at $x = 4$?

- $f(x) = \dfrac{1}{x-4}$
- $f(x) = \dfrac{x^2-16}{x-4}$
- $f(x) = \dfrac{x+4}{x^2-16}$
- $f(x) = \dfrac{x+4}{x-4}$
- $f(x) = x-4$
- $f(x) = \dfrac{x}{4-x}$

HOLE ⇒ Factors to cancel. →

Hole @ $x=4$

$x-4=0$

byebye

~~A~~ B C ~~D~~ ~~E~~ ~~F~~

B) $f(x) = \dfrac{x^2-16}{x-4} = \dfrac{(\cancel{x-4})(x+4)}{\cancel{x-4}} = x+4$

c) $f(x) = \dfrac{x+4}{x^2-16} = \dfrac{\cancel{x+4}}{(x-4)(\cancel{x+4})} = \dfrac{1}{x-4}$

Question 23 of 54

Given $y = \left(\dfrac{x}{3} + \dfrac{6}{x}\right)^3$, find $y'(-3)$.

- $\frac{1}{3}$
- -9
- -27
- $-\frac{2}{3}$
- 0
- 3

$$y = \left(\frac{x}{3} + \frac{6}{x}\right)^3$$

$$y = \left(\frac{x}{3} + 6x^{-1}\right)^3$$

$$y' = 3\left(\frac{x}{3} + 6x^{-1}\right)^2 \frac{d}{dx}\left(\frac{x}{3} + 6x^{-1}\right)$$

$$= 3\left(\frac{x}{3} + \frac{6}{x}\right)^2 \cdot \left(\frac{1}{3} - 6x^{-2}\right)$$

$$= 3\left(\frac{x}{3} + \frac{6}{x}\right)^2 \left(\frac{1}{3} - \frac{6}{x^2}\right)$$

$$y'(-3) = 3\left(\frac{-3}{3} + \frac{6}{-3}\right)^2\left(\frac{1}{3} - \frac{6}{(-3)^2}\right)$$

$$= 3(-1-2)^2\left(\frac{1}{3} - \frac{6}{9}\right)$$

$$= 3(-3)^2\left(\frac{1}{3} - \frac{2}{3}\right)$$

$$= 3 \cdot 9 \left(\frac{-1}{3}\right) = -9$$

Question 32 of 54

Which of the following is equal to $\cos x$?

- $\lim\limits_{h\to 0} \dfrac{\sin(x+h) - \sin x}{h}$
- $-\sin x$

$$\cos(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$\parallel$

$f'(x)$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

- $\lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$
- $-\sin x$
- $\lim_{x\to 0} \frac{\cos(x+h)-\cos(x)}{h}$
- $\lim_{h\to 0} \frac{\sin(x+h)+\sin x}{h}$
- $\lim_{x\to 0} \frac{\sin(x+h)-\sin x}{h}$
- $\lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h}$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

$= \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$

A

Question 35 of 54

Find $y'(x)$.

$$y = \frac{3}{(4-x^2)^2}$$

- $\frac{-12x}{4-x^2}$
- $\frac{-6}{4-x^2}$
- $\frac{-6}{(4-x^2)^3}$
- $\frac{12x}{4-x^2}$
- $\frac{12x}{(4-x^2)^3}$
- $\frac{-12x}{(4-x^2)^3}$

$y = \frac{3}{(4-x^2)^2}$

Rewrite $y = 3(4-x^2)^{-2}$

Chain Rule

$y = 3(-2)(4-x^2)^{-3}\frac{d}{dx}(4-x^2)$

$= -6(4-x^2)^{-3}(-2x)$

$= 12x(4-x^2)^{-3}$

$= \frac{12x}{(4-x^2)^3}$

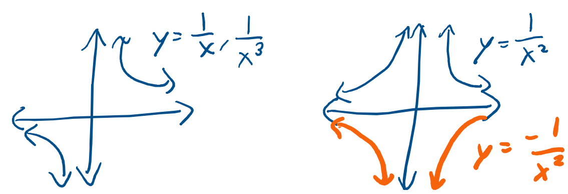
E

Question 36 of 54

Which of the following statements is **TRUE**:

- $\lim_{x\to 0} \frac{5}{x} = 5$
- $\lim_{x\to 8} \frac{1}{x-8} = \infty$
- $\lim_{x\to -3} \frac{-1}{x+3} = +\infty$
- $\lim_{x\to 1} \frac{-7}{(x-1)^3} = -\infty$
- $\lim_{x\to 0} \frac{1}{x^2} = 0$
- $\lim_{x\to \frac{1}{2}} \frac{-3}{(2x-1)^2} = -\infty$

~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~ F



Question 39 of 54

Choose the number of correct statements regarding the piecewise function below.

$$f(x) = \begin{cases} 6x^2 + 9 & x \le 0 \\ 13x + 9 & 0 < x < 1 \\ -13x + 9 & x \ge 1 \end{cases}$$

I. $\lim_{x\to 0} f(x) = 9.$

II. $\lim_{x\to 1^-} f(x) = 22.$

III. $\lim_{x\to 1^+} f(x) = -4.$

IV. $f(x)$ has a hole at $x = 0$.

V. $f(x)$ has a jump at $x = 1$.

- Only one statement is true.
- None of the statements is true.
- All five statements are true.
- Only four statements are true.
- Only two statements are true.
- Only three statements are true.

$6x^2+9$ | $13x+9$ | $-13x+9$

0 | 1

I $\lim_{x\to 0^-}(6x^2+9) \stackrel{?}{=} \lim_{x\to 0^+}(13x+9)$

$9 = 9$ ✓

II $\lim_{x\to 1^-}(13x+9) = 13+9 = 22$

III $\lim_{x\to 1^+}(-13x+9) = -13+9 = -4$

IV $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \neq f(0)$

$9 = 9 \stackrel{?}{=} f(0)$

V $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ ✓

By II & III

0