1 derivative of ln

1 derivative of product rule

1 acceleration

1 implicit

2 related rates

1 critical pt

1 relative extrema

1 increasing/decreasing

1 inflection

1 combined
(inc/dec) + (concavity)

1 abs max/min

Question 4 of 54

A spherical balloon is being deflated at a rate of $\frac{13}{2}$ cubic centimeters per minute. How fast is the radius of the ballon changing at the instant when the radius is 6 centimeters? The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

- $\frac{126}{13\pi}$ cm/min
- $\frac{13}{1152\pi}$ cm/min
- $\frac{13\pi}{96}$ cm/min
- $\frac{13}{288\pi}$ cm/min
- $\frac{96\pi}{13}$ cm/min
- $\frac{169}{96\pi}$ cm/min

$$\frac{dV}{dt} = -\frac{13}{2}\ \frac{cm^3}{s}$$

$$\left.\frac{dr}{dt}\right|_{r=6}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$-\frac{13}{2} = 4\pi(6)^2\frac{dr}{dt}$$

$$-\frac{13}{2} \cdot \frac{1}{4\pi(36)} = \frac{dr}{dt}$$

$$\begin{array}{r} 4 \\ 36 \\ \times 8 \\ \hline 288 \end{array}$$

~~A~~ B ~~C~~, D ~~E~~ ~~F~~

Question 2 of 54

What is the slope of the tangent line to the graph of $y = \ln(2x^3 + 5x)$ at $x = 1$?

- $\frac{1}{7}$
- 77
- ln 7
- $\frac{11}{7}$
- $\frac{7}{11}$
- $\frac{1}{2}$

$$y = \ln(2x^3+5x) \text{ @ } x=1$$

$$y' = \frac{1}{2x^3+5x}\ \frac{d}{dx}(2x^3+5x)$$

$$= \frac{1}{2x^3+5x}(6x^2+5)$$

$$= \frac{1}{2+5} \cdot (6+5)$$

$$= \frac{11}{7} \quad D$$

Question 5 of 54

Which of the following numbers IS a critical number of the function $g(x) = \frac{1}{6}x^6 + \frac{11}{5}x^5 + 7x^4$?

- 7
- −2
- −4
- 5
- −6
- 3

$g(x) = \frac{1}{6}x^6 + \frac{11}{5}x^5 + 7x^4$

$g'(x) = \frac{6}{6}x^5 + \frac{11}{5} \cdot 5x^4 + 28x^3$

$= x^5 + 11x^4 + 28x^3$

$= x^3(x^2 + 11x + 28)$

$= x^3(x+7)(x+4) = 0$

$x = 0, -7, -4$

C

Question 9 of 54

Suppose the derivative of $f(x)$ is $f'(x) = x^3(x+2)(x-4)$. At what x-value(s) does $f(x)$ have a local maximum?

- $x = 4$
- $x = 0$
- $x = 0$ and $x = 4$
- $x = -2$
- $x = -2$ and $x = 4$
- $x = -2$ and $x = 0$

$f'(x) = x^3(x+2)(x-4) = 0$

$x = 0, -2, 4$

$-$ $+$ $-$ $+$

-3 -1 1 5

-2 0 4 f'

$+ \cdot + \cdot +$

$x = 0$

B

Question 11 of 54

Find the x values at which the inflection points of $f(x) = \frac{1}{6}x^4 + x^3 - 10x^2 + 11$ occur.

- $x = 2$ and $x = 3$
- $x = -5$ and $x = -2$
- $x = 2$ and $x = 5$
- $x = 0$ and $x = 5$
- $x = 0$ and $x = 2$
- $x = -5$ and $x = 2$

11 $f(x) = \frac{1}{6}x^4 + x^3 - 10x^2 + 11$

$f'(x) = \frac{4}{6}x^3 + 3x^2 - 20x$

$f''(x) = \frac{4 \cdot 3}{6}x^2 + 6x - 20$

$= 2x^2 + 6x - 20$

$= 2(x^2 + 3x - 10)$

$= 2(x+5)(x-2) = 0$

$x = -5, 2$

~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~ F

$+$ $-$ $+$

-6 0 3

-5 2 f''

Question 23 of 54

The derivative of a function, $g(x)$, is given by

$$g'(x) = e^x(x+2)^2(x^2 - 5)$$

Find the largest open interval(s) on which the function $g(x)$ is increasing.

- $(-2, \sqrt{5})$
- $(-\infty, -2)$ and $(\sqrt{5}, \infty)$
- $(-\sqrt{5}, 0)$ and $(\sqrt{5}, \infty)$
- $(-\infty, -\sqrt{5})$ and $(\sqrt{5}, \infty)$
- $(-\infty, -\sqrt{5})$ and $(0, \sqrt{5})$
- $(-\sqrt{5}, \sqrt{5})$

$g'(x) = \cancel{e^x}(\cancel{x+2})^2(x^2 - 5) = 0$

$x = -2, \pm\sqrt{5}$

$+$ $-$ $-$ $+$

-3 -2.1 0 3

$-\sqrt{5}$ -2 $\sqrt{5}$

$(-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ D

○ $(-\infty, -\sqrt{5})$ and $(0, \sqrt{5})$

○ $(-\sqrt{5}, \sqrt{5})$

Question 14 of 54

Consider the function

$$f(x) = \frac{1}{3}x^3 - 7x^2 + 13x + 98.6.$$

On which interval is the graph of $f(x)$ both decreasing and concave down?

○ $(7, \infty)$

○ $(1, 13)$

○ $(7, 13)$

○ $(-\infty, 1)$

○ $(-\infty, 7)$

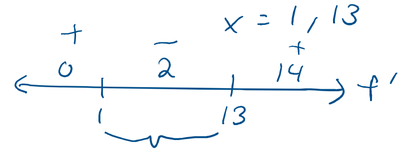
○ $(1, 7)$

(14) $f(x) = \frac{1}{3}x^3 - 7x^2 + 13x + 98.6$

$f'(x) = x^2 - 14x + 13 = 0$

$(x-13)(x-1) = 0$

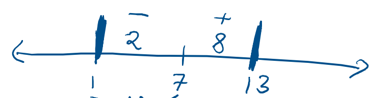
$x = 1, 13$



Decreasing: $(1, 13)$

$f''(x) = 2x - 14 = 0$

$x = 7$



Concave Down

Ans: $(1, 7)$ (F)

Question 10 of 54

A plane flies due east at a speed of 390 miles per hour at an altitude of 5 miles. It passes directly over a radar station on the ground. How fast is the distance between the plane and the radar station changing after the plane has travelled 12 miles?

○ 395 mi/h

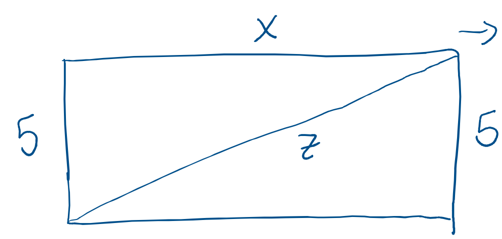
○ 400 mi/h

○ 402 mi/h

○ 375 mi/h

○ 360 mi/h

○ 423 mi/h



→ 390 miles per hr

$\frac{dx}{dt} = 390 \qquad \left.\frac{dz}{dt}\right|_{x=12} = ?$

$x^2 + 5^2 = z^2$

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

$\cancel{2}(12)(390) = \cancel{2}z\frac{dz}{dt}$

$12(390) = 13\frac{dz}{dt}$

$360 = \frac{12(\cancel{390}^{30})}{\cancel{13}} = \frac{dz}{dt}$

Find z with $x^2 + 5^2 = z^2$ and $x = 12$

$12^2 + 5^2 = z^2$

$144 + 25 = z^2$

$169 = z^2$

$z = 13$

Question 32 of 54

$f(x)$ is a polynomial and

critical pts

$$f'(1) = 0, \qquad f'(4) = 0$$

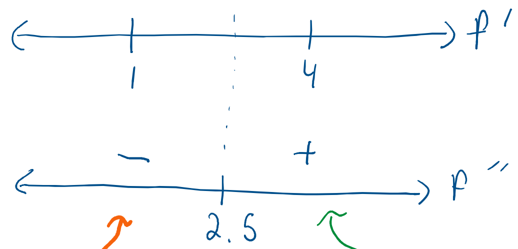
$$f''(2.5) = 0, \qquad f''(x) < 0 \text{ for } x < 2.5, \qquad f''(x) > 0 \text{ for } x > 2.5$$

Not enough info.

Which of the following statements are true?

I. $(1, f(1))$ is an inflection point of $f(x)$.
II. $(2.5, f(2.5))$ is an inflection point of $f(x)$. ✓ b/c
III. $f(x)$ has a relative maximum at x=1. ✓
IV. $f(x)$ has a relative minimum at x=4. ✓

- (selected) Only II, III and IV are true.
- Only II and III are true.
- Only I, II and IV are true.
- Only I and IV are true.
- Only II and IV are true.
- Only I and III are true.



By 2^{nd} Derivative Test
$f''(1) < 0 \Rightarrow \cap \Rightarrow$ rel max

By 2^{nd} Derivative
$f''(4) > 0 \Rightarrow \cup \Rightarrow$ rel min

Question 50 of 54

Given $f(x) = \ln \sqrt[3]{\frac{3+3x}{3-x}}$, find $f'(1)$.

- $\frac{1}{8}$
- $\frac{1}{3}$
- $\frac{1}{2}$
- $\frac{1}{6}$
- $\frac{1}{4}$
- $\frac{2}{3}$

Rewrite $y = \ln \sqrt[3]{\frac{3+3x}{3-x}}$

$$= \ln\left(\frac{3+3x}{3-x}\right)^{1/3}$$

$$= \frac{1}{3}\ln\left(\frac{3+3x}{3-x}\right)$$

$$= \frac{1}{3}\left[\ln(3+3x) - \ln(3-x)\right]$$

$$y' = \frac{1}{3}\left[\frac{1}{3+3x}\frac{d}{dx}(3+3x) - \frac{1}{3-x}\cdot\frac{d}{dx}(3-x)\right]$$

$$y' = \frac{1}{3}\left[\frac{3}{3+3x} - \frac{-1}{3-x}\right]$$

$$y'(1) = \frac{1}{3}\left[\frac{3}{3+3} + \frac{1}{3-1}\right] = \frac{1}{3}\left[\frac{1}{2} + \frac{1}{2}\right]$$

$$= \frac{1}{3}$$