Practice "Exam 4" Set

Question 1 of 54

Calculate $\int_1^5 (x^2 - 3x + 2)\,dx$.

- $\frac{38}{3}$
- 16
- $-\frac{75}{2}$
- $\frac{85}{6}$
- $\frac{40}{3}$
- 24

$$\int x^n dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$\int_1^5 x^2 - 3x + 2 \, dx$$

$$= \frac{x^3}{3} - \frac{3x^2}{2} + 2x \Big]_1^5$$

$$= \frac{(5)^3}{3} - \frac{3 \cdot 5^2}{2} + 2(5) - \left(\frac{1}{3} - \frac{3}{2} + 2\right)$$

Question 2 of 54

Given

$$\int_{-1}^{1} 3f(x)\ dx = 3, \qquad \int_{-1}^{3} 2f(x)\ dx = 6 \qquad \text{and} \qquad \int_{1}^{3} [f(x) + g(x)]\ dx = 2,$$

Compute

$$\int_1^3 g(x)\ dx.$$

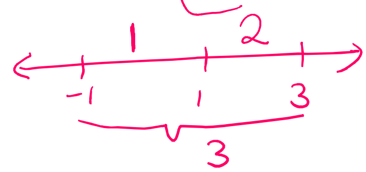
- 1
- 5
- 3
- 0
- −1
- −2

$$\int_{-1}^{1} 3f(x)dx = 3 \Rightarrow \int_{-1}^{1} f(x)dx = 1$$

$$\int_{-1}^{3} 2f(x)dx = 6 \Rightarrow \int_{-1}^{3} f(x)dx = 3$$

$$\int_1^3 f(x) + g(x)\,dx = 2$$

$$\underbrace{\int_1^3 f(x)dx}_{2} + \boxed{\int_1^3 g(x)dx} = 2$$



$$2 + \int_1^3 g(x)dx = 2$$

$$\int_1^3 g(x)dx = 0$$

Question 4 of 54

Use the Trapezoidal Rule to approximate the integral $\int_1^4 e^{(x^2-1)}\,dx$ with 3 trapezoids.

- $T_3 = e^3 + e^8 + e^{15}$
- $T_3 = 1 + e^3 + e^8 + e^{15}$
- $T_3 = e^3 + e^8 + \frac{1}{2}e^{15}$
- $T_3 = \frac{1}{2} + 2e^3 + 2e^8 + 2e^{15}$
- $T_3 = \frac{1}{2} + e^3 + e^8 + \frac{1}{2}e^{15}$
- $T_3 = \frac{1}{4} + \frac{1}{2}e^3 + \frac{1}{2}e^8 + \frac{1}{4}e^{15}$

$$\Delta x = \frac{\overset{a}{4} - \overset{b}{1}}{\underset{n}{3}} = \frac{3}{3} = 1$$

	1	2	3	4
e^{x^2-1}	e^{1-1}	e^{4-1}	e^{9-1}	e^{16-1}

$\underbrace{e^{4-1} \quad e^{9-1}}_{\times 2}$

$$T_3 = \left(1 + 2e^3 + 2e^8 + e^{15}\right)\frac{\Delta x}{2}$$

$T_3 = \left(1 + 2e^3 + 2e^8 + e^{15}\right)\frac{\Delta x}{2}$

$= \frac{1}{2} + e^3 + e^8 + \frac{1}{2}e^{15}$

Question 8 of 54

Given $\frac{dy}{dt} = \frac{2}{3}y$ and $y(0) = 18$. Find $y(t)$.

- ○ $18e^{3t}$
- ● $18e^{\frac{2}{3}t}$
- ○ $\frac{2}{3}e^{18t}$
- ○ $18e^{2t}$
- ○ $9e^{\frac{3}{2}t}$
- ○ $9e^{3t}$

$\frac{dy}{dt} = \frac{2}{3}y \Rightarrow y = Ce^{2/3 t}$

$\frac{dy}{dt} = ky \Rightarrow y = Ce^{kt}$

$y(0) = 18 \Rightarrow 18 = Ce^{2/3(0)}$

$C = 18 \Rightarrow y = 18e^{2/3 t}$

Question 10 of 54

Evaluate

$$\int_1^4 \frac{x-5}{\sqrt{x}}\,dx.$$

- ○ 18
- ○ $\frac{7}{2}$
- ○ $\frac{18}{5}$
- ○ $-\frac{15}{4}$
- ● $-\frac{16}{3}$
- ○ -24

$\int_1^4 \frac{x}{x^{1/2}} - \frac{5}{x^{1/2}}\,dx$

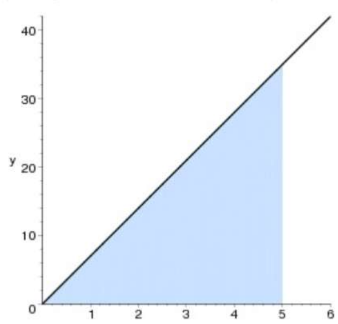
$= \int_1^4 x^{1/2} - 5x^{-1/2}\,dx$

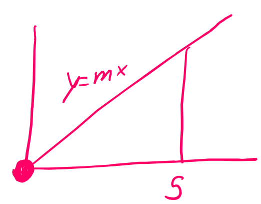
$= \frac{2}{3}x^{3/2} - 5 \cdot \frac{2}{1}x^{1/2} \Big]_1^4$

$= \frac{16}{3} - 20 - \left(\frac{2}{3} + 10\right)$

Question 12 of 54

Which of the following definite integrals represents the area of the shaded region?





C (D)

- ○ $\int_0^5 35\,dx$
- ○ $\int_0^5 7\,dx$
- ○ $\int_0^5 x\,dx$
- ● $\int_0^5 7x\,dx$
- ○ $\int_0^5 (7x + 35)\,dx$

- ○ $\int_0^5 7\,dx$
- ○ $\int_0^5 x\,dx$
- ● $\int_0^5 7x\,dx$
- ○ $\int_0^5 (7x+35)\,dx$
- ○ $\int_0^5 (x+35)\,dx$

Question 17 of 54

Find the area of the region bounded by the graphs of the following equations: $y = \left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)^2$, $y = 0$, $x = 1$ and $x = e$.

- ● $2e + \frac{1}{2}e^2 - \frac{3}{2}$
- ○ $2\sqrt{e} + \frac{2}{3}\sqrt{e^3} - \frac{8}{3}$
- ○ $\frac{1}{2}e^2 + \frac{1}{2}$
- ○ $\frac{\sqrt{e^3}}{3} + \frac{1}{3\sqrt{e^3}} + \sqrt{e} + \frac{1}{\sqrt{e}} - \frac{8}{3}$
- ○ $\frac{7}{2} + 2e + \frac{1}{2}e^2$
- ○ $\frac{7}{2} - 2e - \frac{1}{2}e^2$

$y = \left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)$, $x = 1, e$

Find the area.

$$\int_1^e \left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)^2 dx$$

$$= \int_1^e \left(\frac{1}{\sqrt{x}}\right)^2 + 2\left(\frac{1}{\sqrt{x}}\right)(\sqrt{x}) + (\sqrt{x})^2\, dx$$

$$= \int_1^e \frac{1}{x} + 2 + x\, dx$$

$$= \ln(x) + 2x + \frac{x^2}{2} \Big]_1^e$$

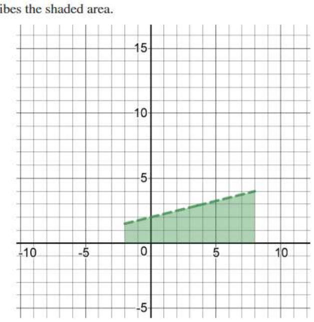
$$= \cancel{\ln(e)}^{1} + 2e + \frac{e^2}{2} - \left(\cancel{\ln(1)}^{0} + 2 + \frac{1}{2}\right)$$

$$= 1 + 2e + \frac{e^2}{2} - \frac{5}{2}$$

$$= 2e + \frac{e^2}{2} - \frac{3}{2}$$

Question 21 of 54

Write a definite integral that describes the shaded area.



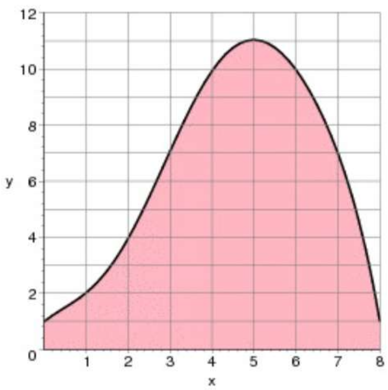
- ○ $\int_0^4 (\frac{2}{3}x)\,dx$
- ○ $\int_0^4 (\frac{2}{3}x+2)\,dx$
- ○ $\int_0^4 (x+2)\,dx$
- ● $\int_{-2}^8 (\frac{1}{4}x+2)\,dx$
- ○ $\int_{-2}^8 (\frac{1}{3}x+2)\,dx$
- ○ $\int_{-2}^8 (4x+2)\,dx$

$\int_{-2}^{8} (mx + 2)dx$ ~~A~~ ~~B~~ ~~C~~ D E F

$m = \frac{4-2}{8-0} = \frac{2}{8} = \frac{1}{4}$ $(0, 2)$ $(8, 4)$

Question 27 of 54

Use the Trapezoidal Rule to approximate the area of the shaded region with $n = 4$.



- ● 50
- ○ 48
- ○ 52
- ○ 49
- ○ 51
- ○ 53

$\Delta x = \frac{8-0}{4} = 2$

x	0	2	4	6	8
$f(x)$	1	4	10	10	1

×2

$T_4 = \frac{\Delta x}{2}\left(1 + 2(4) + 2(10) + 2(10) + 1\right)$

2

$= 1 + 8 + 20 + 20 + 1 = 50$

Question 41 of 54

Let T be the Trapezoidal Rule approximation of $\int_3^6 x^2\, dx$ using $n = 3$ trapezoids and E be the exact value of $\int_3^6 x^2\, dx$. What is $T - E$?

- ○ 2.5
- ○ 1
- ○ 1.5
- ● 0.5
- ○ 2
- ○ 3

$\Delta x = \frac{6-3}{3} = 1$

	3	4	5	6
x^2	9	16	25	36

×2

$T = \frac{\Delta x}{2}(9 + 32 + 50 + 36)$

$= \frac{1}{2}(9 + 32 + 50 + 36)$

$E = \int_3^6 x^2\, dx = \left.\frac{x^3}{3}\right]_3^6$

$= \frac{6^3}{3} - \frac{3^3}{3}$

$T - E = \frac{1}{2}(9 + 32 + 50 + 36) - \left(\frac{6^3}{3} - \frac{3^3}{3}\right)$

Question 47 of 54

Given $\int_{-3}^{2} f(x)\,dx = 10$, $\int_{-6}^{2} f(x)\,dx = 15$ and $\int_{-3}^{2} 3g(x)\,dx = 9$. Which of the following is/are true?

I. $\int_{-3}^{2} 5g(x)\,dx = 45$

II. $\int_{-6}^{-3} f(x)\,dx = 5$

III. $\int_{-3}^{2} [2f(x) + g(x)]\,dx = 23$

- I only
- I and III only
- III only
- II and III only
- II only
- I and II only

$$\int_{-3}^{2} f(x)\,dx = 10$$

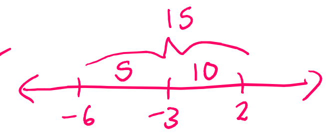
$$\int_{-6}^{2} f(x)\,dx = 15$$

$$\int_{-3}^{2} 3g(x)\,dx = 9 \Rightarrow \int_{-3}^{2} g(x)\,dx = 3$$

$$\cancel{\mathrm{I}} \quad \frac{\int_{-3}^{2} 5g(x)\,dx}{5} = \frac{45}{5}$$

$$\int_{-3}^{2} g(x)\,dx = 9$$

$$\mathrm{II} \quad \int_{-6}^{-3} f(x)\,dx = 5 \checkmark$$



$$\mathrm{III} \quad \int_{-3}^{2} 2f(x) + g(x)\,dx = 23 \checkmark$$

$$2\int_{-3}^{2} f(x)\,dx + \int_{-3}^{2} g(x)\,dx$$

$$= 2 \cdot 10 + 3 = 23$$