Lesson 11: The Chain Rule Pt 2
Derivative of Natural Logarithmic Function

Recall when $y = f(g(x))$ then $y' = f'(g(x)) \cdot g'(x)$

Trig & Exponential Functions w/ chain Rule

Note $\sin^2 x = (\sin x)^2$ ← Check by plugging $x = \frac{\pi}{2}$ into both. Are they equal?

$\neq$

$\sin(x^2)$

Ex 1: Find y' when $y = \sin(x^2)$

Let $f(x) = \sin(x)$ $\quad g(x) = x^2$

$f'(x) = \cos(x)$ $\quad g'(x) = 2x$

By chain rule,

$$y' = f'(g(x)) \cdot g'(x)$$
$$= f'(x^2) \cdot 2x$$
$$= \cos(x^2) \cdot 2x$$
$$= 2x\cos(x^2)$$

Ex 2: Find y' when $y = \sec(-2x+1)\tan(3x)$

Product Rule: $u(x) = \sec(-2x+1)$ $\quad v(x) = \tan(3x)$

↳ Chain Rule $\quad$ ↳ Chain Rule

$u'(x) = \sec(-2x+1)\tan(-2x+1)(-2)$ $\quad v'(x) = \sec^2(3x) \cdot 3$

By product rule,

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$$y' = u'v + uv'$$

$$= \sec(-2x+1)\tan(-2x+1)(-2)\tan(3x)$$

$$+ \sec(-2x+1)\sec^2(3x)\cdot 3$$

$$= -2\sec(-2x+1)\tan(-2x+1)\tan(3x)$$

$$+ 3\sec(-2x+1)\sec^2(3x)$$

<u>Ex 3</u>: Find y' when $y = e^{(1-2x)^4}$

By Chain Rule,

$$y' = e^{(1-2x)^4}\frac{d}{dx}\left[(1-2x)^4\right]$$

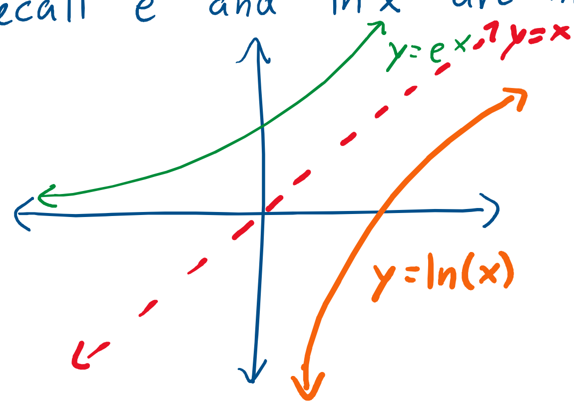
$$= e^{(1-2x)^4}\cdot 4(1-2x)^3\frac{d}{dx}\left[1-2x\right]$$

$$= e^{(1-2x)^4}\cdot 4(1-2x)^3(-2)$$

$$= -8(1-2x)^3 e^{(1-2x)^4}$$

Derivative of Logarithmic Functions

Recall e^x and $\ln x$ are inverses. B/c they are inverses



$$\text{Slope of } \ln(x) = \frac{1}{\text{Slope of } e^x}$$

So for $x > 0$,

$$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}$$

Ex 4: Find y' when $y = x\ln(x)$

Product Rule: $u = x$ $\quad v = \ln(x)$

$u' = 1$ $\quad v' = \frac{1}{x}$

So $y' = u'v + uv'$

$= 1 \cdot \ln(x) + x \cdot \frac{1}{x}$

$= \ln(x) + 1$

Ex 5: Find y' when $y = \ln(3x^2 + x + 1)$

Chain Rule

$$y' = \frac{1}{3x^2+x+1} \frac{d}{dx}(3x^2+x+1)$$

$$= \frac{1}{3x^2+x+1} \cdot (6x+1)$$

$$= \frac{6x+1}{3x^2+x+1}$$

Recall

$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

$\ln(xy) = \ln(x) + \ln(y)$

$\ln(x^m) = m\ln(x)$

$\ln(1) = 0$

$\ln(e^x) = x$

Ex 6: Find y' when $y = \ln \sqrt[3]{\frac{x^2+1}{2x-1}}$

Rewrite y with Logarithmic Rules to avoid double chain w/ quotient rule

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$$y = \ln\left[\left(\frac{x^2+1}{2x-1}\right)^{1/3}\right] = \frac{1}{3}\ln\left(\frac{x^2+1}{2x-1}\right) = \frac{1}{3}\left[\ln(x^2+1) - \ln(2x-1)\right]$$

$$= \frac{1}{3}\ln(x^2+1) - \frac{1}{3}\ln(2x-1)$$

By chain rule,

$$y' = \frac{1}{3}\cdot\frac{1}{x^2+1}\frac{d}{dx}(x^2+1) - \frac{1}{3}\cdot\frac{1}{2x-1}\cdot\frac{d}{dx}(2x-1)$$

$$= \frac{1}{3}\cdot\frac{1}{x^2+1}\cdot 2x - \frac{1}{3}\cdot\frac{1}{2x-1}\cdot 2$$

$$= \frac{2x}{3(x^2+1)} - \frac{2}{3(2x-1)}$$

Ex 6: Find y' when $y = \ln(\sin(x)\cdot e^{2x})$

Rewrite to avoid chain rule w/ product rule.

$$y = \ln(\sin(x)) + \ln(e^{2x})$$

$$= \ln(\sin(x)) + 2x$$

By chain rule,

$$y' = \frac{1}{\sin(x)}\frac{d}{dx}(\sin(x)) + 2$$

$$= \frac{1}{\sin(x)}\cdot\cos(x) + 2$$

$$= \frac{\cos(x)}{\sin(x)} + 2$$

$$= \cot(x) + 2$$

Ex 7: Find y' of $y = \ln\left(\frac{x^{3/2}(x+1)}{(x+3)^2}\right)$

Rewrite
$$y = \ln\left(x^{3/2}(x+1)\right) - \ln\left[(x+3)^2\right]$$
$$= \ln\left(x^{3/2}\right) + \ln(x+1) - \ln\left[(x+3)^2\right]$$
$$= \frac{3}{2}\ln(x) + \ln(x+1) - 2\ln(x+3)$$

By chain rule,
$$y' = \frac{3}{2} \cdot \frac{1}{x} + \frac{1}{x+1} \cdot \cancelto{1}{\frac{d}{dx}(x+1)} - \frac{2}{x+3}\cancelto{1}{\frac{d}{dx}(x+3)}$$
$$= \frac{3}{2} \cdot \frac{1}{x} + \frac{1}{x+1} - \frac{2}{x+3}$$