# Lesson 12: Higher Order Derivatives

The derivative of a function, $f(x)$, is also called first derivative,

$$y', \; f'(x), \; \frac{dy}{dx}, \; \frac{d}{dx}[f(x)]$$

If we take the derivative of the first derivative of $y=f(x)$, then we get second derivative

$$y'', \; f''(x), \; \frac{d^2y}{dx^2}, \; \frac{d^2}{dx^2}[f(x)]$$

Take the derivative n times, we get the nth derivative.

$$y^{(n)}, \; f^{(n)}(x), \; \frac{d^ny}{dx^n}, \; \frac{d^n}{dx^n}[f(x)]$$

Ex 1: Find the first three derivatives of

$$R(x) = 3x^2 + 8x^{1/2} + e^x$$

① $R'(x) = 3(2)x + 8\left(\frac{1}{2}\right)x^{-1/2} + e^x$

$= 6x + 4x^{-1/2} + e^x$

② $R''(x) = \frac{d}{dx}\left[6x + 4x^{-1/2} + e^x\right]$

$= 6 + 4\left(-\frac{1}{2}\right)x^{-3/2} + e^x$

$= 6 - 2x^{-3/2} + e^x$

③ $R'''(x) = \frac{d}{dx}\left[6 - 2x^{-3/2} + e^x\right]$

$= 0 - 2\left(\frac{-3}{2}\right)x^{-5/2} + e^x$

$= 3x^{-5/2} + e^x$

Ex 2: Find the first five derivatives of $y = \sin(x)$.

Recall $\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$

Derivatives of $\sin x$ & $\cos x$

**Derivatives of sin x & cos x**

$$\begin{array}{ccc} & \sin x & \\ \nearrow & & \searrow \\ -\cos x & & \cos x \\ \nwarrow & & \swarrow \\ & -\sin x & \end{array}$$

① $y' = \frac{d}{dx}[\sin x] = \cos(x)$

② $y'' = \frac{d}{dx}[\cos(x)] = -\sin(x)$

③ $y''' = \frac{d}{dx}[-\sin x] = -\frac{d}{dx}[\sin(x)] = -\cos(x)$

④ $y^{(4)} = \frac{d}{dx}[-\cos(x)] = -\frac{d}{dx}[\cos(x)] = -(-\sin(x)) = \sin(x)$

⑤ $y^{(5)} = \frac{d}{dx}[\sin(x)] = \cos(x)$

Note if $y = \cos x$, a similar pattern occurs

Ex 3: Given $f(x) = xe^x$. Find $f''(x)$ and $f^{(3)}(x)$.

① Find $f'$.

Product Rule $\quad u = x \quad v = e^x$

$\quad u' = 1 \quad v' = e^x$

$$\begin{aligned} f'(x) &= u'v + uv' \\ &= 1 \cdot e^x + xe^x \\ &= (1+x)e^x \end{aligned}$$

② Find $f''$.

Product Rule: $\quad u = 1+x \quad v = e^x$

$\quad u' = 1 \quad v' = e^x$

$$\begin{aligned} f''(x) &= u'v + uv' \\ &= 1 \cdot e^x + (1+x)e^x \\ &= (1+1+x)e^x \\ &= (2+x)e^x \end{aligned}$$

③ Find $f'''$.

Product Rule: $u = 2+x \quad v = e^x$

$u' = 1 \quad v' = e^x$

$$f'''(x) = u'v + uv'$$
$$= 1e^x + (2+x)e^x$$
$$= (1+2+x)e^x$$
$$= (3+x)e^x$$

Ex 4: Find $f''(x)$ of $f(x) = \dfrac{4}{(x^2+1)^2}$

Rewrite $f(x) = 4(x^2+1)^{-2}$

① Find $f'$.

By Chain Rule,

$$f'(x) = 4(-2)(x^2+1)^{-3}(2x)$$
$$= \frac{-16x}{(x^2+1)^3}$$

② Find $f''$.

By Quotient Rule,

$u = -16x \quad v = (x^2+1)^3$

$u' = -16 \quad v' = 3(x^2+1)^2(2x)$

$$f''(x) = \frac{u'v - uv'}{v^2} = \frac{-16(x^2+1)^3 - (-16x)(3)(x^2+1)^2(2x)}{((x^2+1)^3)^2}$$
$$= \frac{(-16)\cancel{(x^2+1)^2}\left[x^2+1 - x(3)(2x)\right]}{(x^2+1)^{\cancel{6}\,4}}$$

$$= \frac{-16(x^2+1-6x^2)}{(x^2+1)^4}$$

$$= \frac{-16(x^{[illegible]}+1-6x^{[illegible]})}{(x^2+1)^4}$$

$$= \frac{-16(-5x^2+1)}{(x^2+1)^4}$$

## Position & Velocity & Acceleration Function

$s(t)$ $v(t)$

Recall that $v(t) = s'(t)$

Acceleration Function $[a(t)]$ tells us how fast the velocity changes

Hence $a(t) = v'(t) = [s'(t)]' = s''(t)$

Ex 5: The position function of a particle is

$$s(t) = \frac{1}{12}t^4 - \frac{4}{3}t^3 + 8t^2 - 64t$$

What is the acceleration of the particle @ $t=2$?

$$a(t) = s''(t) \quad @ \ t=2$$

$$s'(t) = \frac{4}{12}t^3 - \frac{4}{3}\cdot 3t^2 + 8\cdot 2t - 64$$

$$= \frac{1}{3}t^3 - 4t^2 + 16t - 64$$

$$s''(t) = \frac{3}{3}t^2 - 4(2)t + 16 + 0$$

$$= t^2 - 8t + 16$$

$$a(2) = s''(2) = 2^2 - 8(2) + 16$$

$$= 4$$