MA 16010 LESSONS 14-15: RELATED RATES

This space is left for you to take your own notes.

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Recipe for Solving a Related Rates Problem

Step 1: Draw a good picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you **WANT** to find.

Step 3: Find an equation relating the relevant variables. This usually involves a formula from geometry, similar triangles, the Pythagorean Theorem, or a formula from trigonometry. **Use your picture!**

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t*.

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you **WANT**. Do **NOT** substitute before this step!

Some Useful Formulas

Right Triangle	Triangle	Trapezoid
Pythagorean Theorem: $$a^2 + b^2 = c^2$$	$$A = \frac{1}{2}bh$$ $$P = a + b + c$$	$$A = \frac{1}{2}(a + b)h$$
Rectangular Box $$V = lwh$$ $$S = 2(hl + lw + hw)$$	Rectangle $$A = lw$$ $$P = 2l + 2w$$	Circle $$A = \pi r^2$$ $$C = 2\pi r$$
Right Circular Cylinder $$V = \pi r^2 h$$ $$SA = 2\pi rh$$	Sphere $$V = \frac{4}{3}\pi r^3$$ $$S = 4\pi r^2$$	Cone $$V = \frac{1}{3}\pi r^2 h$$ $$SA = \pi rl + \pi r^2$$

Example 1: If x and y are both functions of t and $x + y^3 = 2$.

(a)   Find $\frac{dy}{dt}$ when $\frac{dx}{dt} = -2$ and $y = 1$.

(b)   Find $\frac{dx}{dt}$ when $\frac{dy}{dt} = 3$ and $x = 1$
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Example 2: A spherical balloon is being deflated at a constant rate of 20 cubic cm per second. How fast is the radius of the balloon changing at the instant when the balloon's radius is 12 cm?

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

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KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t*.

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

Example 3: A ladder 5 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.3 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 3 m from the wall?

> **Step 1:** Draw a picture. Label all constant values and give variable names to any changing quantities.

> **Step 2:** Determine what information you **KNOW** and what you WANT to find.

> **KNOW:**   **WANT:**

> **Step 3:** Find an equation relating the relevant variables.

> **Step 4:** Use implicit differentiation to differentiate the equation with respect to time t.

> **Step 5:** Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

Example 4: A plane is flying directly away from you at 500 mph at an altitude of 3 miles.

  (1) How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you?

      Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

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            KNOW: **WANT:**

      Step 3: Find an equation relating the relevant variables.

      Step 4: Use implicit differentiation to differentiate the equation with respect to time t.

      Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

Example 4: A plane is flying directly away from you at 500 mph at an altitude of 3 miles.

(2) How fast is the angle of elevation changing when it is $\pi/3$?

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you WANT to find.

KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

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Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT.

HW 15.5: A cylindrical tank standing upright (with one circular base on the ground) has a radius of 22 cm for the base. How fast does the water level in the tank drop when the water is being drained at 28 cm³/*sec*? Note: The formula right circular cylinder is $V = \pi r^2 h$.

      Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

      Step 2: Determine what information you **KNOW** and what you WANT to find.

         KNOW: **WANT:**

      Step 3: Find an equation relating the relevant variables.

      Step 4: Use implicit differentiation to differentiate the equation with respect to time *t*.

      Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

HW 16.3: A baseball diamond is a square 90 ft on a side. A player runs from first base to second base at 14 ft/sec. At what rate is the player's distance from home base increasing when he is halfway from first to second base?

    Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

    Step 2: Determine what information you **KNOW** and what you WANT to find.

        KNOW: **WANT:**

    Step 3: Find an equation relating the relevant variables.

    Step 4: Use implicit differentiation to differentiate the equation with respect to time t.

    Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**