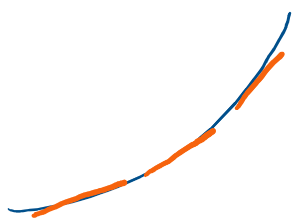
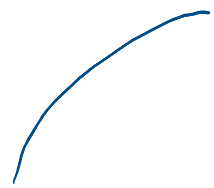
Lesson 17: Increasing and Decreasing Functions
First Derivative Test

A function is increasing if the function value gets bigger and bigger.

A function is decreasing if the function value gets smaller and smaller.





Notice that slope of tangent lines when $f(x)$

increasing $\Rightarrow$ positive

decreasing $\Rightarrow$ negative

Thm: Let $f(x)$ be a continuous and differentiable function on an open interval, I.

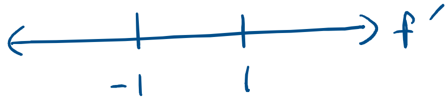
- If $f'(x) > 0$ for all x in I, then $f(x)$ is increasing
- If $f'(x) < 0$ for all x in I, then $f(x)$ is decreasing

Ex 1: Given $f(x) = x^3 - 3x$. Find where f is inc/dec.

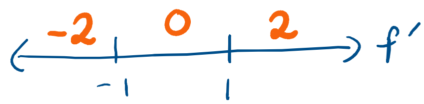
Step 1: Find when $f'(x) = 0$.

$$f'(x) = 3x^2 - 3 = 0$$
$$3(x^2 - 1) = 0$$
$$3(x-1)(x+1) = 0$$
$$x = \pm 1$$

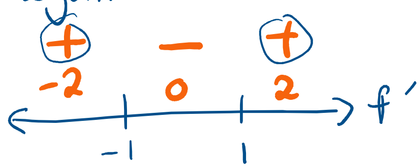
Step 2: Draw a # line with points from Step 1.



Step 3: Determine test points for the # line.



Step 4: Plug these test points into f′ to determine whether it's positive or negative.



$f'(x) = 3(x-1)(x+1)$

$f'(-2) = 3 \cdot - \cdot - = +$

$f'(0) = 3 \cdot - \cdot + = -$

$f'(2) = 3 \cdot + \cdot + = +$

Step 5: Use definition of increasing/decreasing

Inc: $(-\infty, -1) \cup (1, \infty)$

Dec: $(-1, 1)$

Ex 2: Given $f(x) = 2x^2 e^{4x+1}$. Find where f is inc/dec.

Find $f'(x) = 0$

By Product Rule, $u = 2x^2 \quad v = e^{4x+1}$

$u' = 4x \quad v' = e^{4x+1} \cdot 4$

$$f'(x) = u'v + uv'$$

$$= 4x \cdot e^{4x+1} + 2x^2 e^{4x+1} \cdot 4 = 0$$

$$4x e^{4x+1} [1 + 2x] = 0$$

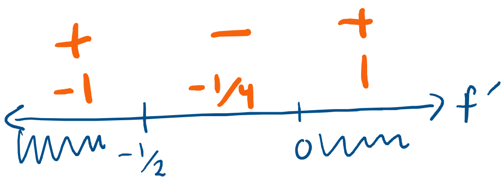
$\downarrow \quad \downarrow \quad \downarrow$

$x = 0$ Never! $x = -\frac{1}{2}$

Note: $e^x > 0$ for all x.

\+ − +

$f'(x) = 4x(\cancel{e^{4x+1}})(1+2x)$



$f'(x) = 4x\cancel{(4x+1)}(1+2x)$

$f'(-1) = - \cdot + \cdot - = +$

$f'(-\frac{1}{4}) = - \cdot + \cdot + = -$

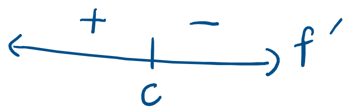
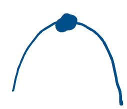
$f'(1) = + \cdot + \cdot + = +$

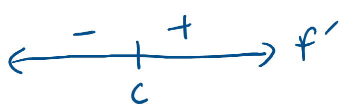
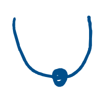
<u>Inc</u>: $(-\infty, -\frac{1}{2}) \cup (0, \infty)$

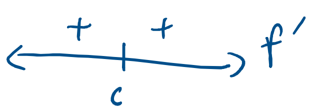
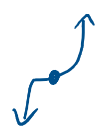
<u>Dec</u>: $(-\frac{1}{2}, 0)$

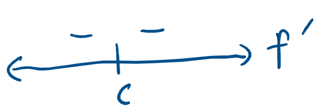
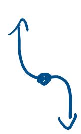
The First Derivative Test

Let c be a critical # of f(x) that is continuous on an open interval, I, containing c.

①  ex.  ⇒ relative max @ $x = c$

②  ex.  ⇒ relative min @ $x = c$

③  ex.  ⇒ neither

④  ex.  ⇒ neither

<u>Ex 3</u>: Given $f(x) = 2x^4 - 2x^3$

ⓐ Find where f is inc/dec.

$$f'(x) = 8x^3 - 6x^2 = 0$$

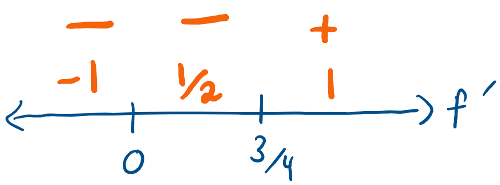
$$2x^2(4x-3) = 0$$

$$x = 0,\ x = 3/4$$

$-$ $-$ $+$

$f'(x) = (\cancel{2x^2})(4x-3)$

$x = 0,\ x = 3/4$



$f'(x) = \cancel{(2x^2)}(4x-3)$

$f'(-1) = + \cdot - = -$

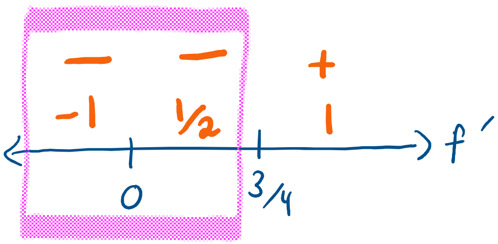
$f'(\tfrac{1}{2}) = + \cdot - = -$

$f'(1) = + \cdot + = +$

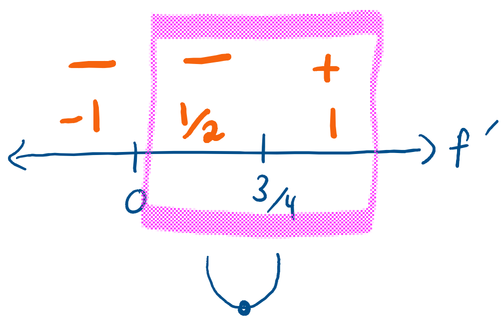
Inc: $(3/4, \infty)$

Dec: $(-\infty, 3/4)$

(b) Find the relative extrema of $f(x)$.



By Case 4, $x=0$ is not relative extrema



By Case 2, $x = 3/4$ is relative min.

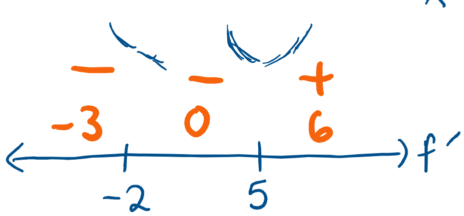
Ex 4: Given $f'(x) = (3x+6)^2(x-5)$

(a) Find where f is inc/dec.

$f'(x) = (3x+6)^2(x-5) = 0$

$3x+6=0 \quad | \quad x-5=0$

$x=-2 \quad | \quad x=5$



$f'(x) = \cancel{(3x+6)^2}(x-5)$

$f'(-3) = + \cdot - = -$

$f'(0) = + \cdot - = -$

$f'(6) = + \cdot + = +$

ⓑ Find the relative extremas.

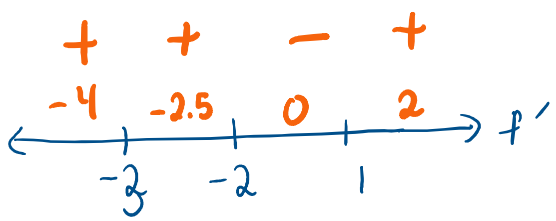
By the # line $x=5$ is a relative min and $x=-2$ is nothing.

Ex 5: Given $f'(x) = (2x+4)(x+3)^2(x-1)$

ⓐ Find where f is inc/dec.

$$f'(x) = (2x+4)(x+3)^2(x-1) = 0$$

$$x = -2, -3, 1$$



$f'(x) = (2x+4) \cdot + \cdot (x-1)$

$f'(-4) = - \cdot + \cdot - = +$

$f'(-2.5) = - \cdot + \cdot - = +$

$f'(0) = + \cdot + \cdot - = -$

$f'(2) = + \cdot + \cdot + = +$

Inc: $(-\infty, -2) \cup (1, \infty)$

Dec: $(-2, 1)$

ⓑ Find relative extrema.

$x = -3$ is nothing

$x = -2$ is relative max

$x = 1$ is relative min