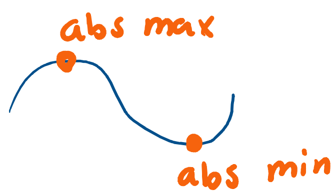
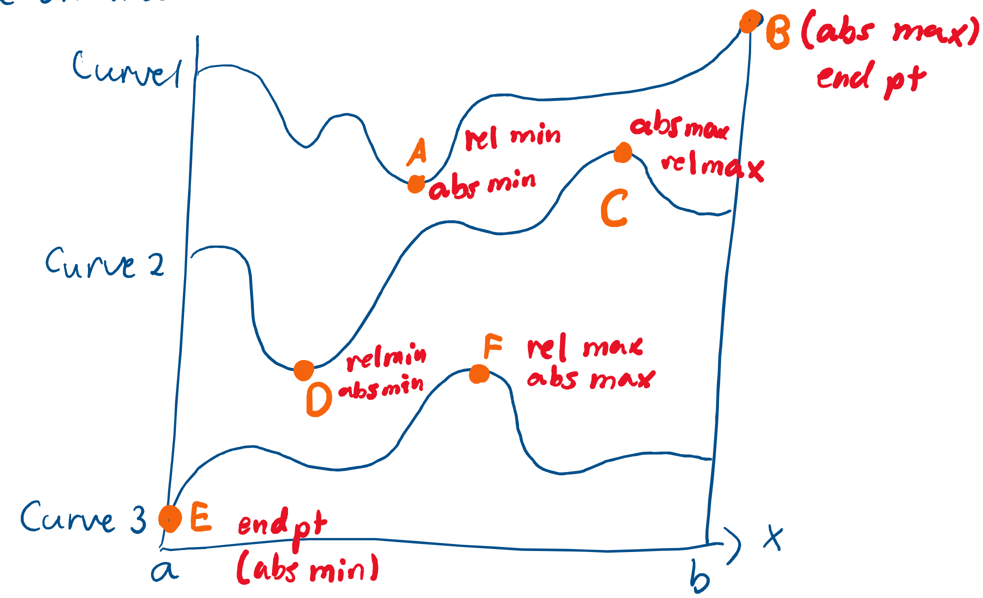
Monday, March 2, 2026   1:26 PM

Lesson 19: Absolute Extrema on an Interval

An absolute max is the largest function value on the entire interval.

An absolute min is the smallest function value on the entire interval.





<u>Thm</u>: If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both an absolute max and min on the interval.

The absolute extrema only occurs either at

① critical number or ② end point

<u>Note</u>: Relative ⟺ Local Extrema
Absolute ⟺ Global Extrema

Steps to Find Absolute Extrema

① Find all critical numbers. $[f'(x)=0]$

② Check if points from ① are in the interval given.

③ Plug ② and endpoints into $f(x)$.

④ Compare the function values and determine absolute extrema.
i.e. Biggest $f(x)$ value is absolute max

(4) Compare the function values and determine absolute extrema.
i.e. Biggest $f(x)$ value is absolute max
Smallest $f(x)$ value is absolute min

Ex 1: Find the abs extrema of
$y = x^4 - 2x^2$ on $[-1, 1]$

Step 1: Find when $f'(x) = 0$
$y' = 4x^3 - 2 \cdot 2x = 0$
$4x^3 - 4x = 0$
$4x(x^2 - 1) = 0$
$4x(x-1)(x+1) = 0$
$x = 0, -1, 1$

Step 2: Check if $-1, 0, 1$ are in $[-1, 1]$? Yes!

Step 3: Plug $x = -1, 0, 1$ and endpoints into $f(x) = x^4 - 2x^2$

x	$y = x^4 - 2x^2$	Conclusion
-1	$(-1)^4 - 2(-1)^2 = 1 - 2 = -1$	Abs min
0	0	Abs max
1	$1^4 - 2(1)^2 = 1 - 2 = -1$	Abs min

Abs min @ $(-1, -1)$
$(1, -1)$

Abs max @ $(0, 0)$

Ex 2: Find the absolute extrema of $y = x^4 - 2x^3$ on $[-1, 1]$

Step 1: Find when $f'(x) = 0$.
$y' = 4x^3 - 2 \cdot 3x^2 = 0$
$2x^2(2x - 3) = 0$
$x = 0, 3/2$

Step 2: Check if $x = 0, 3/2$ are in $[-1, 1]$?
Only $\underline{0}$ is in $[-1, 1]$

Step 3: Plug $x = 0$ and endpts into $f(x)$

x	$y = x^4 - 2x^3$	Conclusion
-1	$(-1)^4 - 2(-1)^3 = 1 + 2 = 3$	Abs max
0	0	
1	$1 - 2 = -1$	Abs min

Abs min @ $(1, -1)$
Abs max @ $(-1, 3)$

Ex 3: Find the absolute extrema of
$y = xe^x$ on $[-2, 0]$

… and endpoints in $f(x)$

$y = xe^x$ on $[-2, 0]$

Step 1: Find $f'(x) = 0$.

$u = x \qquad v = e^x$

$u' = 1 \qquad v' = e^x$

By product rule,

$f'(x) = 1 \cdot e^x + x \cdot e^x = 0$

$e^x(1 + x) = 0$

Never $= 0$

$x = -1$

Step 2: Check if $x = -1$ is in $[-2, 0]$? Yes.

Step 3: Plug $x = -1$ and endpoints in $f(x)$.

x	$y = xe^x$	Conclusion
-2	$-2e^{-2} = -2/e^2$	Abs min
-1	$-e^{-1} = -1/e = -e/e^2$	
0	0	Abs max

Abs min $\left(-2, \frac{-2}{e^2}\right)$

Abs max $(0, 0)$

Ex 4: Find the absolute extrema of $y = \frac{6x^2}{x+1}$ on $(-1, 5]$?

Step 1: Find when $f'(x) = 0$.

$u = 6x^2 \qquad v = x + 1$

$u' = 12x \qquad v' = 1$

By quotient rule

$$y' = \frac{12x(x+1) - 1(6x^2)}{(x+1)^2} = 0$$

$$\frac{12x^2 + 12x - 6x^2}{(x+1)^2} = 0$$

$$\frac{6x^2 + 12x}{(x+1)^2} = 0$$

$= 0 \leftarrow 6x^2 + 12x$

DNE $\leftarrow (x+1)^2$

$6x^2 + 12x = 0$

$6x(x + 2) = 0$

$x = 0, -2$

$\cancel{(x+1)^2 = 0}$

$\cancel{x = -1}$

$\Downarrow$

This isn't in my domain.

Why? B/c original function is $\frac{6x^2}{x+1}$

Step 2: Is $x = 0, -2$ in $(-1, 5]$?

Only $x = 0$

Step 3: Plug $x = 0$ and endpoints (only w/ bracket)

x	$y = 6x^2/(x+1)$	Conclusion
0	$\frac{6 \cdot 0}{1} = 0$	Abs min
5	$\frac{6 \cdot 5^2}{6} = 25$	Abs max

Abs min $(0, 0)$

Abs max $(5, 25)$

Ex 5: Find the absolute extrema of

my domain. function is $x+1$

Ex 5: Find the absolute extrema of

$$y = -x^2 - 2x \text{ on } (-2, 0)$$

Step 1: Find when $f'(x) = 0$,

$$y' = -2x - 2 = 0$$

$$x = -1$$

Step 2: Check if -1 is in $(-2, 0)$?

Yes!

Step 3: Plug $x = -1$ and the endpoints in $f(x)$

x	$y = -x^2 - 2x$
-1	$-(-1)^2 - 2(-1) = -1 + 2 = 1$

To determine max or min you need either 1st or 2nd derivative test.

1st Derivative Test

$+ \quad \cap \quad -$

$-2 \qquad 0$

f'

-1

By the test, rel max

OR

2nd Derivative test

$$y'' = -2$$

$\cap$

$$y''(-1) = -2 < 0 \Rightarrow \text{rel max}$$