# Lesson 21 : Limits at Infinity Pt 2

## Slant Asymptotes

The line $y = ax + b$, where a and b are constants and $a \neq 0$, is a slant asymptote ot $f(x)$ if $f(x)$ gets closer and closer to $y = ax + b$ as $x \to \pm \infty$.

To find the slant asymptotes of a rational function, we use
- Synthetic Division
- Long Division

## Synthetic Division Slide 1

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division
To use Synthetic Division, find the zeros of the denominator of y.
  i.e. $x - 2 = 0 \implies x = 2$

  2 |

# Synthetic Division Slide 2

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division
Next, list the coefficients of the polynomial on the numerator.

$$2 \,\big|\; 1 \quad 2 \quad 8$$

# Synthetic Division Slide 3

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division
Drag down the first # inside the ⌞

$$2 \,\big|\; 1 \quad 2 \quad 8$$

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# Synthetic Division Slide 4

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 1: Synthetic Division</u>

Multiply 2 by the value just written on the bottom row.

```
2 | 1    2    8
  |   ↓    2
  ------------
    1
```

# Synthetic Division Slide 5

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 1: Synthetic Division</u>

Add the column created.

```
2 | 1    2 |   8
  |   ↓    2 ↓
  ------------
    1    4
```

# Synthetic Division Slide 6

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Repeat: Multiply 2 by the value just written on the bottom row.

$$
\begin{array}{c|ccc}
2 & 1 & 2 & 8 \\
  & \downarrow & 2 & 8 \\
\hline
  & 1 & 4 &
\end{array}
$$

# Synthetic Division Slide 7

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Repeat: Add the column created.

$$
\begin{array}{c|ccc}
2 & 1 & 2 & 8 \\
  & \downarrow & 2 & 8 \\
\hline
  & 1 & 4 & 16
\end{array}
$$

# Synthetic Division Slide 8

Ex5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Rewrite   $y = \dfrac{x^2 + 2x + 8}{x - 2} = x + 4 + \dfrac{16}{x - 2}$

$$
\begin{array}{c|ccc}
2 & 1 & 2 & 8 \\
  & \downarrow & 2 & 8 \\
\hline
  & 1 & 4 & 16 \\
\end{array}
$$

←remainder

# Synthetic Division Slide 9

Ex5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

Method 1: Synthetic Division

Rewrite   $y = \dfrac{x^2 + 2x + 8}{x - 2} = \boxed{x + 4} + \dfrac{16}{x - 2}$

⇓

Slant Asymptote

i.e. Slant Asymptote $y = x + 4$

# Long Division Slide 1

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

**Method 2: Long Division**
Long Division works w/o knowing the zeros of $y$'s denominator. But involves a lot of work.

$$x - 2 \, \overline{\smash{)}x^2 + 2x + 8}$$

Divisor          Dividend

# Long Division Slide 2

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

**Method 2: Long Division**
Divide the first term of the dividend by first term of the divisor.

$$x - 2 \, \overline{\smash{)}x^2 + 2x + 8}$$

① $\dfrac{x^2}{x} = x$

# Long Division Slide 3

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

**Method 2: Long Division**

Place what's found in ① above √

$$x - 2 \,\, \overline{\smash{\big)}\, x^2 + 2x + 8} \quad\quad\quad ① \,\, \dfrac{x^2}{x} = x$$
$$\phantom{x-2 \,\,\big)\,} x$$

# Long Division Slide 4

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

**Method 2: Long Division**

Multiply that term in ① with divisor.

$$x - 2 \,\, \overline{\smash{\big)}\, x^2 + 2x + 8} \quad\quad ① \,\, \dfrac{x^2}{x} = x$$
$$\phantom{x-2 \,\,\big)\,} x \quad\quad\quad\quad\quad ② \,\, x(x - 2) = x^2 - 2x$$

# Long Division Slide 5

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

**Method 2: Long Division**

Line up what you found in ② under the dividend

$$
\begin{array}{r}
x \phantom{xxxxxxxx} \\
x - 2 \enclose{longdiv}{x^2 + 2x + 8} \\
\underline{-(x^2 - 2x)} \phantom{xx}
\end{array}
$$

① $\dfrac{x^2}{x} = x$

② $x(x-2) = x^2 - 2x$

# Long Division Slide 6

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

**Method 2: Long Division**

Subtract and bring down the next term.

$$
\begin{array}{r}
x \phantom{xxxxxxxx} \\
x - 2 \enclose{longdiv}{x^2 + 2x + 8} \\
\underline{-(x^2 - 2x)} \downarrow \\
4x + 8
\end{array}
$$

① $\dfrac{x^2}{x} = x$

② $x(x-2) = x^2 - 2x$

# Long Division Slide 7

Ex5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 2: Long Division</u>

Repeat: Divide the first term of the dividend by first term of the divisor.

$$x - 2 \, \overline{\smash{)}\, x^2 + 2x + 8}$$
quotient: $x$
$-(x^2 - 2x)$
$4x + 8$

① $\dfrac{4x}{x} = 4$

# Long Division Slide 8

Ex5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 2: Long Division</u>

Repeat: Place what's found in ① above $\sqrt{\phantom{x}}$

$$x - 2 \, \overline{\smash{)}\, x^2 + 2x + 8}$$
quotient: $x + 4$
$-(x^2 - 2x)$
$4x + 8$

① $\dfrac{4x}{x} = 4$

# Long Division Slide 9

Ex5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 2: Long Division</u>

Repeat: Multiply that term in ① with divisor.

$$
\begin{array}{r}
x + 4 \\
x - 2 \overline{\smash{)}\, x^2 + 2x + 8} \\
-(x^2 - 2x) \\
\hline
4x + 8
\end{array}
$$

① $\dfrac{4x}{x} = 4$

② $4(x - 2) = 4x - 8$

# Long Division Slide 10

Ex5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 2: Long Division</u>

Repeat: Line up what you found in ② under the dividend

$$
\begin{array}{r}
x + 4 \\
x - 2 \overline{\smash{)}\, x^2 + 2x + 8} \\
-(x^2 - 2x) \\
\hline
4x + 8 \\
-(4x - 8)
\end{array}
$$

① $\dfrac{4x}{x} = 4$

② $4(x - 2) = 4x - 8$

# Long Division Slide 11

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 2: Long Division</u>
Repeat: Subtract.

$$
\begin{array}{r}
x + 4 \\
x - 2 \enclose{longdiv}{x^2 + 2x + 8} \\
-(x^2 - 2x) \phantom{xxx} \downarrow \\
\hline
4x + 8 \\
-(4x - 8) \\
\hline
16
\end{array}
$$

① $\dfrac{4x}{x} = 4$

② $4(x - 2) = 4x - 8$

# Long Division Slide 12

Ex 5: Find the slant asymptote of $y = \dfrac{x^2 + 2x + 8}{x - 2}$

<u>Method 2: Long Division</u>
Rewrite $y = \dfrac{x^2 + 2x + 8}{x - 2}$

$= \boxed{x + 4} + \dfrac{16}{x - 2}$

⇓

Slant Asymptote

i.e. Slant Asymptote $y = x + 4$

$$
\begin{array}{r}
x + 4 \\
x - 2 \enclose{longdiv}{x^2 + 2x + 8} \\
-(x^2 - 2x) \phantom{xxx} \downarrow \\
\hline
4x + 8 \\
-(4x - 8) \\
\hline
16
\end{array}
$$

Ex 6: Find the slant asymptote of

$$y = \frac{x^3 + x^2 + 1}{x + 3} = \frac{x^3 + x^2 + 0x + 1}{x + 3}$$

Synthetic Division: zeros of $x+3 \Rightarrow x = -3$

$$\begin{array}{c|cccc}
-3 & 1 & 1 & 0 & 1 \\
& \downarrow & -3 & 6 & -18 \\
\hline
& 1 & -2 & 6 & -17
\end{array}$$

$$g(x) = \boxed{x^2 - 2x + 6} - \frac{17}{x+3}$$

But g(x) has no slant asymptote

Recall a slant asymptote has the form $y = ax + b$.
Is $y = x^2 - 2x + 6$ of that form? No
Hence no slant asymptote.

Note: You can avoid all that work by checking the difference b/w the power of the leading terms of numerator and denominator.
- If difference $= 0$, then there horizontal asymptote
- If difference $= 1$, then there slant asymptote
- If difference $> 1$, then nothing