Monday, March 30, 2026 1:15 PM

Lesson 27: Antiderivatives and Indefinite Integration

Consider the equation $F'(x) = f(x)$

Two ways to interpret this:

① $f(x)$ derivative of $F(x)$

② $F(x)$ antiderivative of $f(x)$

Notation: $F(x) = \int f(x)\,dx$

Extra Credit 3 Due Wednesday

Friday Virtual Class

With antiderivatives start with $f(x)$ and find $F(x)$.

Ex 1: ⓐ Differentiate $F(x) = x^2 + 2$

$F'(x) = \boxed{2x}$

ⓑ Find $\int 2x\,dx$

What function $F(x)$ has $2x$ as its derivative?

By part ⓐ, one such $F(x)$ is $x^2 + 2$.

But so are:

- x^2
- $x^2 - 1234$
- $x^2 + \text{Constant}$.

Why? Derivative of a constant is zero.

To account for this, use C as an arbitrary constant.

$$\int 2x\,dx = x^2 + C$$

Process of finding all the antiderivatives of a function is called indefinite integration.

Denoted by $\int f(x)\,dx = F(x) + C$ where C is a constant

n "integral of $f(x)$ with respect to x"

Defined by

Read as "integral of $f(x)$ with respect to x"

- $\int$ integral sign
- $f(x)$ integrand
- x integration variable
- C constant of integration

Differentiation Rule	Integration Rule
$\frac{d}{dx}(c) = 0$	$\int 0\,dx = C$
$\frac{d}{dx}(kx) = k$	$\int k\,dx = kx + C$
$\frac{d}{dx}(kf(x)) = kf'(x)$	$\int kf'(x)\,dx = kf(x) + C$
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int nx^{n-1}\,dx = x^n + C$
$\frac{d}{dx}(x^{n+1}) = (n+1)x^n$	$\int (n+1)x^n\,dx = x^{n+1} + C$ $(n+1)\int x^n\,dx = x^{n+1} + C$ $\frac{(n+1)\int x^n\,dx}{(n+1)} = \frac{x^{n+1} + C}{(n+1)}$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Ex 1: Find the indefinite integral

$$\int \left(x^2 + 2\sqrt{x^1}\right)dx = \int \left(x^2 + 2x^{1/2}\right)dx$$

$$= \frac{x^{2+1}}{2+1} + 2\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{x^3}{3} + \frac{2x^{3/2}}{3/2} + C$$

$= \frac{x^3}{3} + \frac{x^{3/2}}{3/2} + C$

$= \frac{x^3}{3} + 2 \cdot \frac{2}{3} x^{3/2} + C = \frac{x^3}{3} + \frac{4}{3} x^{3/2} + C$

Ex 2: Find the indefinite integral

$$\int 3x^5 + \frac{1}{x^2}\, dx = \int (3x^5 + x^{-2})\, dx$$

$$= \frac{3x^{5+1}}{5+1} + \frac{x^{-2+1}}{-2+1} + C$$

$$= \frac{3x^6}{6} + \frac{x^{-1}}{-1} + C$$

$$= \frac{x^6}{2} - \frac{1}{x} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

Ex 3: Find the indefinite integral

$$\int \frac{x^2 + x^{0.5}}{x}\, dx = \int \frac{x^2}{x} + \frac{x^{0.5}}{x}\, dx$$

$$= \int (x + x^{-0.5})\, dx$$

$$= \frac{x^{1+1}}{1+1} + \frac{x^{-0.5+1}}{-0.5+1} + C$$

$$= \frac{x^2}{2} + \frac{x^{0.5}}{0.5} + C$$

$$= \frac{x^2}{2} + 2x^{0.5} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

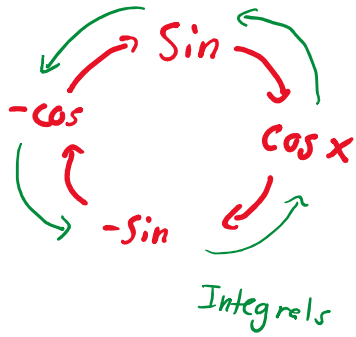
$$\frac{1}{0.5} = \frac{1}{1/2} = 2$$

Differentiate Rule	Integration Rule
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin(x) + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int -\sin x\,dx = \cos(x) + C$ $\Leftrightarrow$ $\int \sin x\,dx = -\cos(x) + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$

Ex 4: Evaluate $\int \frac{\sin x + \cos x}{2}\,dx$

$= \frac{1}{2}\int [\sin(x) + \cos(x)]\,dx$ By Table.

$= \frac{1}{2}[-\cos(x) + \sin(x)] + C$



Note you can always check your answer by taking the derivative of your final answer.

$\frac{d}{dx}\left(\frac{1}{2}[-\cos(x) + \sin(x)] + C\right) = \frac{1}{2}(\sin x + \cos x) + 0$ ✓

Ex 5: Find $\int 5\sec^2 x + \sec x \tan x\,dx$

$= 5\tan(x) + \sec(x) + C$

$= 5\tan(x) + \sec(x) + C$

Differentiation Rule	Integration Rule
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$ ★ This is the exceptian to power rule.

$\int x^{-1} dx = \frac{x^{-1+1}}{-1+1} = \frac{x^0}{0}$

Ex 6: Find $\int x^e + \frac{1}{x} + e^x + 1 \, dx$

$= \int x^e dx + \int \frac{1}{x} dx + \int e^x dx + \int 1 \, dx$

$= \frac{x^{e+1}}{e+1} + \ln\lvert x\rvert + e^x + x + C$

b/c e is #

Ex 7: Find $\int 3x^2 \cos(x^3) dx$ ✓

(A) $y = \sin(x^3)$ ⟶ $y' = \cos(x^3) \cdot \frac{d}{dx}(x^3)$

$= \cos(x^3) \cdot 3x^2$ ✓

(B) $y = \cos(x^3)$

(C) $y = 6x\cos(x^2)$

(D) $y = 6x\sin(x^2)$

(E) $y = x^3\cos(x^3)$