Monday, April 6, 2026 1:29 PM

Lesson 29: Definite Integrals

When the # rectangles used gets bigger and bigger, the approximation gets better.

i.e. The approximations gets closer to the exact signed area.

What happens $n \to \infty$?

Left/Right Riemann Sum approaches the actual signed area.

$$\text{Signed Area} = \int_a^b f(x)\,dx$$

a - lower limit
b - upper limit
of integration

An integral with lower and upper limits is called a definite integral.

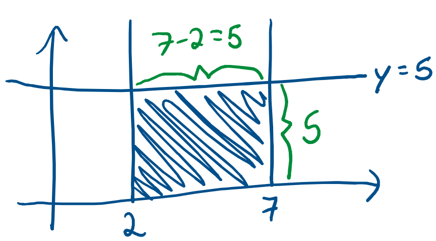
So no more $+C$, when you see lower/upper limits (i.e. a and b)

Ex 1: Evaluate the definite integral $\int_2^7 5\,dx$ by using geometric formula

$5 \downarrow f(x)$

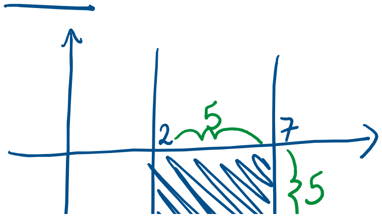
$y = 5$

$\begin{cases} x = 2 \\ x = 7 \end{cases}$



$$\text{Area} = 5 \cdot 5 = 25 = \int_2^7 5\,dx$$

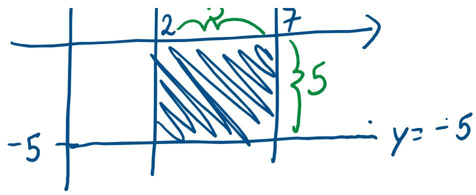
Ex 2: Evaluate the definite integral $\int_2^7 -5\,dx$ by using geometric formula.



$\text{Area} = 5 \cdot 5 = 25$

BUT it's below the x-axis.

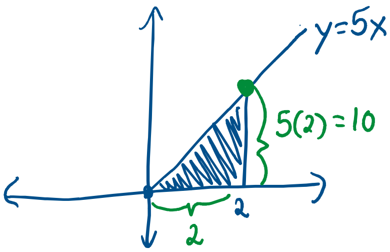
So $\int_2^7 -5\,dx = -25$



BUT it's below the x-axis.

So $\int_2^7 -5\,dx = -25$

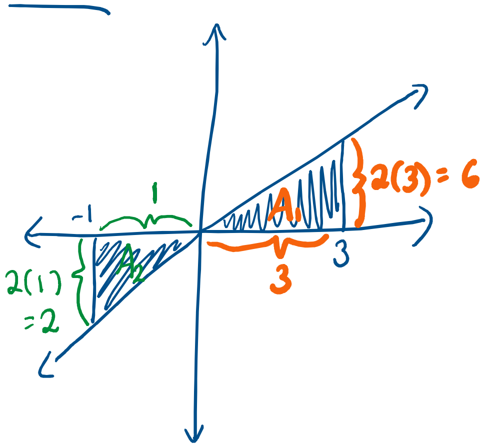
Ex 3: Evaluate the definite integral $\int_0^2 5x\,dx$ by using geometric formula



$\text{Area} = \frac{1}{2}(b)(h) = \frac{1}{2}(2)(10) = 10$

$\int_0^2 5x\,dx = 10$

Ex 4: Evaluate the definite integral $\int_{-1}^{3} 2x\,dx$ by using geometric formulas.

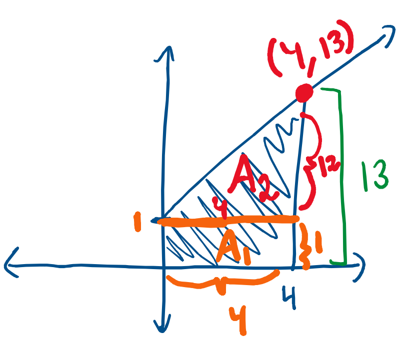


$A_1 = \frac{1}{2}(3)(6) = 9$

$A_2 = \frac{1}{2}(1)(2) = 1$

$\int_{-1}^{2} 2x\,dx = A_1 - A_2 = 9 - 1 = 8$

Ex 5: Evaluate the definite integral $\int_0^4 (3x+1)\,dx$ by using geometric formula



Rectangle

↳ $A_1 = 1 \cdot 4 = 4$

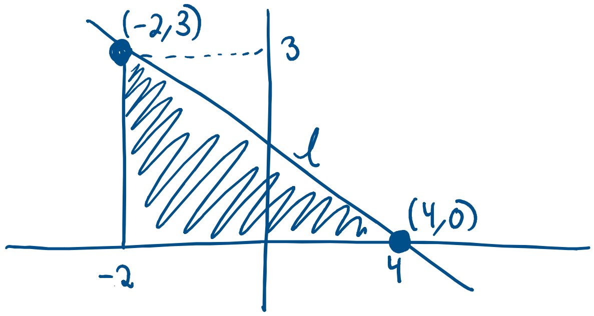
$A_2 = \frac{1}{2}(4)(12) = 24$

$\int_0^4 (3x+1)\,dx = A_1 + A_2 = 4 + 24 = 28$

Ex 6: Write the definite integral that represents the shaded area

$\int^4$

Ex 6: Write the definite integral that represents [illegible] below.



$$\int_{-2}^{4} \boxed{l}\, dx$$

$$m = \frac{0-3}{4-(-2)} = \frac{-3}{6} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + b$$

With $(4,0)$ find b.

$$0 = -\frac{1}{2}(4) + b$$

$$0 = -2 + b$$

$$2 = b$$

$$y = -\frac{1}{2}x + 2$$

Answer: $\int_{-2}^{4} \left(-\frac{1}{2}x + 2\right) dx$