Lesson 30: Definite Integrals Pt 2

Properties of Definite Integrals

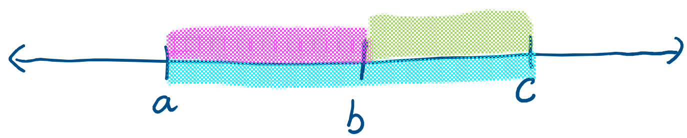
$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)$$

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b \left[f(x) \pm g(x)\right] dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



Ex 1: Given $\int_1^3 f(x)\,dx = 5$, $\int_3^4 f(x)\,dx = 2$ and $\int_1^3 g(x)\,dx = 10$. Evaluate

(a) $\int_1^3 2f(x)\,dx = 2\underbrace{\int_1^3 f(x)\,dx}_{=5 \text{ from the problem statement}} = 2(5) = 10$

(b) $\int_4^3 f(x)\,dx = -\underbrace{\int_3^4 f(x)\,dx}_{=2 \text{ from the problem statement}} = -2$

(c) $\int_1^3 \left[2f(x) - 3g(x)\right] dx = 2\underbrace{\int_1^3 f(x)\,dx}_{=5} - 3\underbrace{\int_1^3 g(x)\,dx}_{=10} = 2(5) - 3(10) = 10 - 30 = -20$

from the problem statement

Ex 1: Given $\int_1^3 f(x)\,dx = 5$, $\int_3^4 f(x)\,dx = 2$ and $\int_1^3 g(x)\,dx = 10$. Evaluate

ⓓ $\int_1^4 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^4 f(x)\,dx = 5+2 = 7$

Ex 2: Given $\int_3^7 x^2\,dx = \frac{316}{3}$, $\int_3^7 x\,dx = 20$, and $\int_3^7 dx = 4$. Evaluate

$$\int_3^7 [-4x^2 + x - 8]\,dx = -4\underbrace{\int_3^7 x^2\,dx}_{316/3} + \underbrace{\int_3^7 x\,dx}_{20} - 8\underbrace{\int_3^7 dx}_{4}$$

$$= -4\left(\frac{316}{3}\right) + 20 - 8(4) = -\frac{1300}{3}$$

Ex 3: Given $\int_0^3 x^2\,dx = 9$, $\int_0^3 x\,dx = \frac{9}{2}$, $\int_0^3 dx = 3$. Evaluate

ⓐ $$\int_0^3 (2x^2 - 3x + 4)\,dx = 2\underbrace{\int_0^3 x^2\,dx}_{9} - 3\underbrace{\int_0^3 x\,dx}_{9/2} + 4\underbrace{\int_0^3 dx}_{3}$$

$$= 2(9) - 3\left(\frac{9}{2}\right) + 4(3) = \frac{33}{2}$$

ⓑ $$\int_0^3 (-x^2 + 5)\,dx = -\underbrace{\int_0^3 x^2\,dx}_{9} + 5\underbrace{\int_0^3 dx}_{3}$$

$$= -9 + 5(3) = 6$$

Ex 4: Given $\int_2^6 2x^3\,dx = 640$. Find

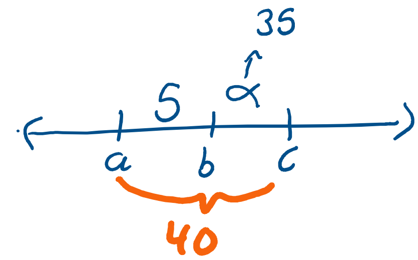
$$\int_2^6 8x^3\,dx = \int_2^6 4\cdot 2x^3\,dx = 4\underbrace{\int_2^6 (2x^3)\,dx}_{640} = 4(640) = 2560$$

640

Ex 5: Given $\int_a^b g(x)\,dx = 5$ and $\int_a^c g(x)\,dx = 8\int_a^b g(x)\,dx$.

Compute $\int_b^c g(x)\,dx = \alpha$

First $\int_a^c g(x)dx = 8\underbrace{\int_a^b g(x)dx}_{=5} = 8(5) = 40$

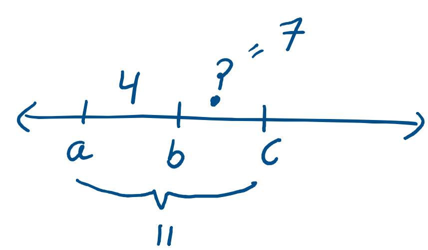


$\int_a^b g(x)dx = 5$

$$\int_b^c g(x)dx = \int_a^c g(x)dx - \int_a^b g(x)dx$$
$$= 40 - 5 = 35$$

Ex 6: Given $\int_a^b f(x)dx = 4$ and $\int_a^c f(x)dx = 11$. Find $\int_b^c f(x)dx$.

$\int_b^c f(x)dx = 11 - 4 = 7$



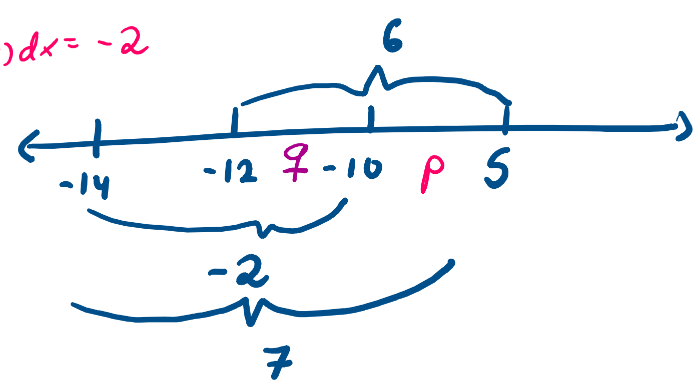
Ex 7: Given $\int_{-12}^{5} f(x)dx = 6$, $\int_{-10}^{-14} f(x)dx = 2$, $\int_{-14}^{5} f(x)dx = 7$

$\Updownarrow$

$\int_{-14}^{-10} f(x)dx = -2$

Find

ⓐ $\int_{-10}^{5} f(x)dx = p = 9$



$$-2 + p = 7$$
$$+2 \qquad +2$$
$$p = 9$$

ⓑ $\int_{5}^{-12} f(x)dx = -\int_{-12}^{5} f(x)dx = -6$

(b) $\int_{5}^{-12} f(x)\,dx = -\int_{-12}^{5} f(x)\,dx = -6$

(c) $\int_{-10}^{-12} f(x)\,dx = -\int_{-12}^{-10} f(x)\,dx = -(-3) = 3$

$$q + p = 6$$
$$q + 9 = 6$$
$$\underline{\quad -9 \quad -9}$$
$$q = -3$$