Lesson 31: The Fundamental Theorem of Calculus (FTC)

Recall

① If we want to find all the antiderivatives of $f(x)$, we evaluate $\int f(x)dx$ [Rules]

② If we want to find signed area under a curve of $f(x)$ from a to b, we evaluate $\int_a^b f(x)dx$ [Pics]

Note both have an integral sign. Can we connect ① and ②?

FTC: Suppose $f(x)$ is continuous on the interval $[a,b]$. If $F(x)$ is an antiderivative of $f(x)$ then

$$\int_a^b f(x)dx = F(b) - F(a)$$

In practice, to integrate $\int_a^b f(x)dx$, we write

$$\int_a^b f(x)dx = F(x)\Big]_a^b = F(b) - F(a)$$

↑ Read as "F(x) evaluated from a to b"

Ex 1: Evaluate $\int_0^3 2x\,dx$

Remember $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$= \frac{2x^{1+1}}{1+1}\Big]_0^3$$

$$= \frac{2x^2}{2}\Big]_0^3$$

$$= x^2\Big]_0^3 = 3^2 - 0^2 = 9$$

Ex 2: Evaluate $\int_0^{\pi/4} \sec^2 x \, dx$ Remember $\int \sec^2 x \, dx = \tan(x)$

$$= \tan(x) \Big]_0^{\pi/4}$$

$$= \tan\left(\frac{\pi}{4}\right) - \tan(0) = 1 - 0 = 1$$

Ex 3: Evaluate $\int_4^9 \frac{2x^3 + \sqrt{x}}{x^2} \, dx$ Remember $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$= \int_4^9 \frac{2x^3}{x^2} + \frac{x^{1/2}}{x^2} \, dx$$

$$= \int_4^9 2x + x^{-3/2} \, dx$$

$$= \frac{2x^2}{2} + \frac{x^{-3/2+1}}{-3/2+1} \Big]_4^9$$

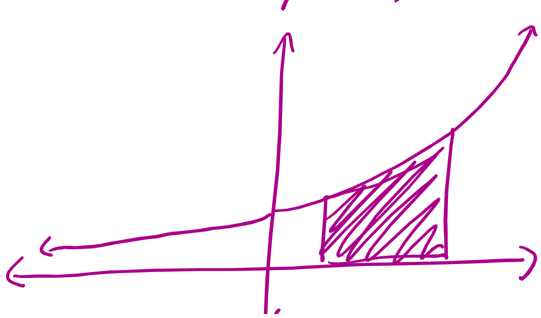
$$= x^2 + \frac{x^{-1/2}}{-1/2} \Big]_4^9$$

$$= \left(x^2 - \frac{2}{\sqrt{x}}\right) \Big]_4^9 = 9^2 - \frac{2}{\sqrt{9}} - \left(4^2 - \frac{2}{\sqrt{4}}\right)$$

$$= 81 - \frac{2}{3} - (16 - 1) = \frac{196}{3}$$

Ex 4: Find the area of the region bounded by the graphs of the following equations:

$y = e^x, y = 0, x = 1,$ and $x = 5$



$$\int_{x=1}^{x=5} e^x \, dx = e^x \Big]_1^5 = e^5 - e^1$$

$$= e^5 - e$$