Lesson 3 2 Fundamental Theorem of Calculus (FTC) pt 2

Recall $\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$

Question: $\int_a^b g'(x)\,dx = ?$ [Does integration undo derivative? No...., not exact]

Well the antiderivative of $g'(x)$ is $g(x)$.

So $\int_a^b g'(x)\,dx = g(x)\Big]_a^b = g(b) - g(a)$

Ex 1: Compute $\int_0^{\pi} (\underbrace{\cos x \tan x}_{?} - \underbrace{\sec^2 x}_{\tan(x)})\,dx$

$= \int_0^{\pi} \left(\cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} - \sec^2 x\right) dx$

$= \int_0^{\pi} (\sin x - \sec^2 x)\,dx$

$= (-\cos x - \tan(x))\Big]_0^{\pi}$

$= -\cos(\pi) - \tan(\pi) - [-\cos(0) - \tan(0)]$

$= -(-1) - 0 - (-1 - 0)$

$= 1 + 1 = 2$

Ex 2: The growth rate of the population of a city is

$$p'(t) = -500(3-t)$$

where t is time in years. How does the population change $t = 1$ year to $t = 3$ years?

$\int_1^3 p'(t)\,dt = \int_1^3 -500(3-t)\,dt$

$= \int^3 -1500 + 500t\,dt$

$$= \int_1^3 -1500 + 500t \, dt$$

$$= \left(-1500t + \frac{500t^2}{2}\right)\Bigg]_1^3$$

$$= \left(-1500t + 250t^2\right)\Bigg]_1^3$$

$$= \left(-1500(3) + 250(3)^2\right) - \left(-1500 + 250\right) = -1000$$

Recall displacement is the difference in position. It could be positive or negative. The sign indicates the direction.

Ex 3: The velocity function, in feet per second, is given for a particle moving along a straight line $v(t) = -10t + 20$ where t is in seconds

(a) Find the displacement from $t=0$ to $t=2$ seconds.

$$\int_0^2 v(t)\,dt = \int_0^2 p'(t)\,dt = \int_0^2 (-10t + 20)\,dt$$

$$= -\frac{10t^2}{2} + 20t \Bigg]_0^2$$

$$= (-5t^2 + 20t)\Bigg]_0^2$$

$$= (-5(2)^2 + 20(2)) - \cancel{(-5(0)^2 + 20(0))}^{\,0}$$

$$= -20 + 40$$

$$= 20$$

(b) Find the displacement from $t=0$ to $t=4$ seconds

$$\int_0^4 v(t)\,dt = \int_0^4 (-10t + 20)\,dt$$

$$= (-5t^2 + 20t)\Bigg]_0^4 \quad \text{by (a)}$$

$$= (-5(4)^2 + 20(4)) - 0$$

$$= -80 + 80$$

$$= 0$$

Ex 4: A faucet is turned on 9:00am and water starts to flow into a tank at a rate of $r(t) = 6\sqrt{t}$ where t is time in hrs after 9am and the rate $r(t)$ is in cubic feet per hr.

(a) How much water, in cubic feet, flows into the tank from 10am to 1pm?

9:00 am $\Rightarrow t=0$
10:00 am $\Rightarrow t=1$
1:00 pm $\Rightarrow t=4$

$$\int_1^4 r(t)\,dt = \int_1^4 6t^{1/2}\,dt$$
$$= \frac{6t^{3/2}}{3/2}\Big]_1^4$$
$$= 6\cdot\frac{2}{3}t^{3/2}\Big]_1^4$$
$$= 4t^{3/2}\Big]_1^4$$
$$= 4(4)^{3/2} - 4(1)^{3/2}$$
$$= 28$$

(b) How many hrs after 9 am will there be 121 cubic feet of water in the tank?

$$\int_0^x r(t)\,dt = 121 \quad \text{Solving for } x.$$
$$\int_0^x 6t^{1/2}\,dt = 4t^{3/2}\Big]_0^x = 4x^{3/2} - \cancel{4(0)^{3/2}}^{\,0} = \frac{4x^{3/2}}{4} = \frac{121}{4}$$
$$x^{3/2} = \frac{121}{4}$$
$$x = \left(\frac{121}{4}\right)^{2/3}$$