Lesson 33: Numerical Integration

Tuesday
5/5 @ 8-10am LD 136

Extra Credit 4 - Attendance
Show up once during review week

With the FTC, we are able to evaluate definite integrals for certain integrands.

However, there many functions that we do not know how to integrate

ex. $f(x) = e^x\sqrt{x^2+1}$ or $f(x) = \frac{\sin x}{x+1}$

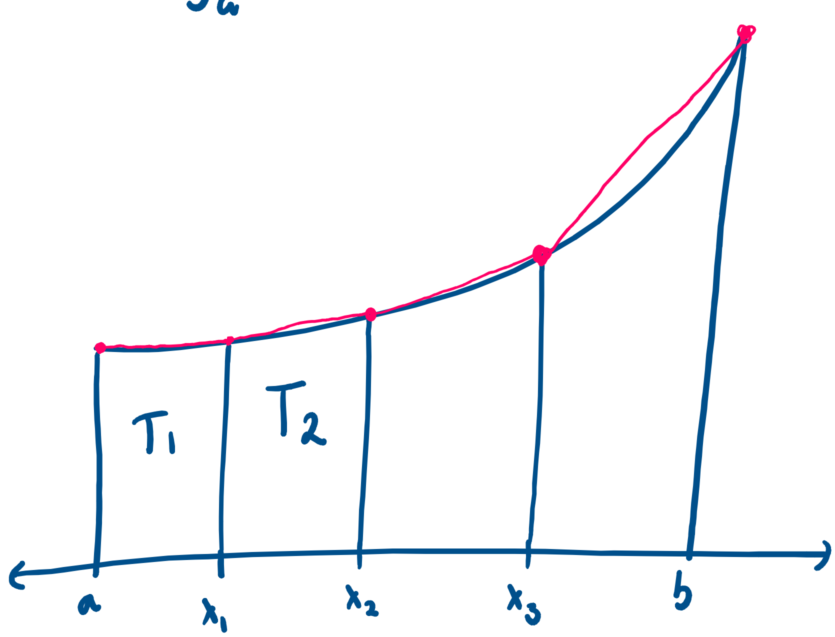
The Trapezoid Rule is an approximation method that allows us to approximate definite integrals.

Trapezoid Rule is similar to Riemann sums.

Instead of using rectangles, we are using trapezoids.

Suppose $f(x)$ is continuous on $[a,b]$. We want to approximate the area

$$\int_a^b f(x)\,dx \text{ using 4 trapezoids.}$$



Recall the area of a trapezoid is

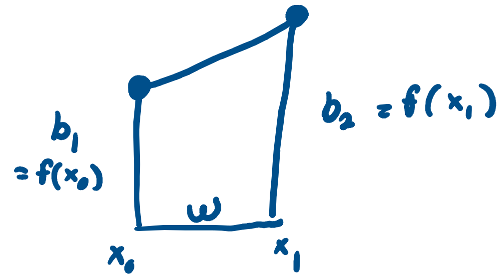
$$A = \frac{1}{2}(b_1 + b_2)w$$

Good news: The width of each sub-interval is the same as in Riemann sums

$$\Delta x = \frac{b-a}{n} = w$$

The area of the first trapezoid is

The bases of the first trapezoid is
$b_1 = f(x_0)$ and $b_2 = f(x_1)$



Area of the 1^{st} trapezoid

$$T_1 = \frac{1}{2}(b_1 + b_2)w = \frac{1}{2}\left(f(x_0) + f(x_1)\right)\Delta x$$

$$T_2 = \quad = \frac{1}{2}\left(f(x_1) + f(x_2)\right)\Delta x$$

$$T_3 = \quad = \frac{1}{2}\left(f(x_2) + f(x_3)\right)\Delta x$$

$$T_4 \quad = \frac{1}{2}\left(f(x_3) + f(x_4)\right)\Delta x$$

Sum all of the trapezoids

$$T_1 + T_2 + T_3 + T_4 = \frac{1}{2}\Delta x\left[f(x_0) + f(x_1) + f(x_1) + f(x_2) + f(x_2) + f(x_3) + f(x_3) + f(x_4)\right]$$

$$= \frac{1}{2}\Delta x\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

We can extend this to n trapezoids

$$T_n = \frac{1}{2}\Delta x\left(f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)\right)$$

where $x_i = a + i\Delta x$

$$\Delta x = \frac{b-a}{n}$$

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MA 16010 LESSON 34: NUMERICAL INTEGRATION (Examples)

EX 1: Use the Trapezoid Rule to approximate $\int_{\underset{a}{0}}^{\overset{b}{3}} x^2 \, dx$ using $n = 3$. Round your answer to the nearest tenth.

Solution: (1) First calculate Δx.

a = 0

b = 3

$b - a$ = 3-0

$\Delta x = \dfrac{b-a}{n}$ = $3/3 = 1$

(2) Determine what $f(x)$ is.

$$\int_0^3 \boxed{x^2} \, dx$$

Hence $f(x) =$ x^2

(3) Find the following values:

x_0 =	0	$f(x_0)$ =	0	$f(x_0)$ =	0
x_1 =	1	$f(x_1)$ =	1	$2 \cdot f(x_1)$ =	2
x_2 =	2	$f(x_2)$ =	4	$2 \cdot f(x_2)$ =	8
x_3 =	3	$f(x_3)$ =	9	$f(x_3)$ =	9

(4) Sum all the values in the black box. = 19

(5) Multiply the value found in (4), Δx found in (1), and 1/2, which yields our answer.

$19 \cdot 1 \cdot \frac{1}{2} = 19/2$

MA 16010 LESSON 34: NUMERICAL INTEGRATION (Examples)

EX 2: Approximate the area of the shaded region by using the Trapezoid Rule with $n = 3$

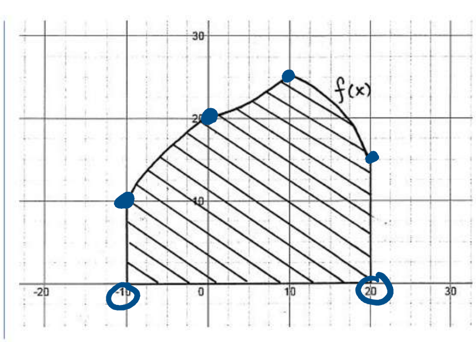
Solution: (1) First calculate Δx.

a = -10

b = 20

$b - a$ = $20-(-10)=30$

$\Delta x = \dfrac{b-a}{n}$ = $\frac{30}{3} = 10$



(2) Find the following values:

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x_0 = -10	$f(x_0)$ = 10	$f(x_0)$ = 10
x_1 = 0	$f(x_1)$ = 20	$2 \cdot f(x_1)$ = 40
x_2 = 10	$f(x_2)$ = 25	$2 \cdot f(x_2)$ = 50
x_3 = 20	$f(x_3)$ = 15	$f(x_3)$ = 15

(3) Sum all the values in the black box. = 115

(4) Multiply the value found in (3), Δx found in (1), and 1/2, which yields our answer.

$115 \times 10 \times \frac{1}{2} = 575$

Ex 3: Given the following data for $f(x)$

x	3	4	5	6	7
f(x)	3	6	-1	20	9

Using the Trapezoid Rule with $n=4$ to estimate the following integral $\int_{3 \to a}^{7 \to b} f(x)\,dx$.

$a = 3$
$b = 7$
$\Delta x = \frac{b-a}{n} = \frac{7-3}{4} = \frac{4}{4} = 1$

	f(x)	
$x_0 = 3$	$f(3) = 3$	3
$x_1 = 4$	$f(4) = 6$	12
$x_2 = 5$	$f(5) = -1$	-2
$x_3 = 6$	$f(6) = 20$	40
$x_4 = 7$	$f(7) = 9$	9
		62

→ multiply it by Δx and $\frac{1}{2}$

$62 \cdot \frac{1}{2} \cdot 1 = 31$

x	3	4	5	6	7
f(x)	3	6	-1	20	9

same double stays the same

$3 + 12 + -2 + 40 + 9 = 62 \left(\frac{1}{2}\right) \underbrace{\Delta x}_{=1} = 62\left(\frac{1}{2}\right) = 31$

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