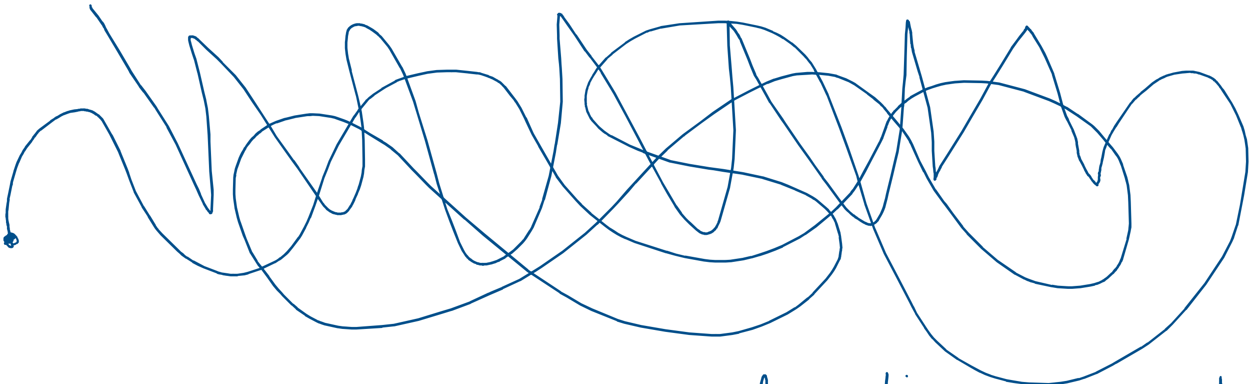
Wednesday, January 21, 2026 1:32 PM

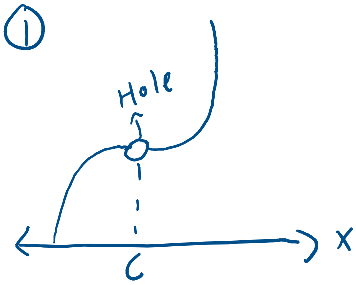
Lesson 4: Continuity



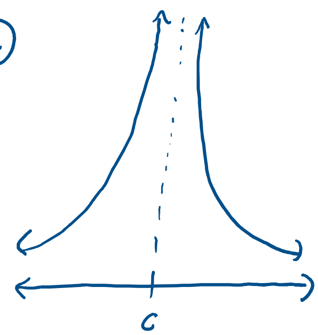
A function is continuous if there is no disruption in the graph.

The following 4 graphs show $f(x)$ is discontinuous @ $x=c$.

(1)

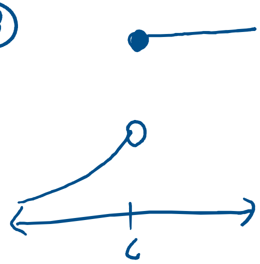


(2)



These 2 graphs have $f(c)$ undefined.

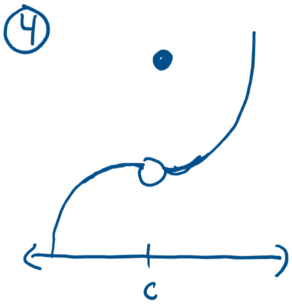
(3)



$f(c)$ is defined

BUT $\lim_{x\to c} f(x) = DNE$

(4)



$f(c)$ is defined **BUT** $\lim_{x\to c} f(x) \neq f(c)$

$\lim_{x\to c} f(c)$ exists

We can see a function $f(x)$ is continuous @ $x=c$ if the following is true:

1. $f(c)$ defined (has a value)
2. $\lim_{x\to c} f(x)$ exists

If any of 3 conditions aren't met, then we say $f(x)$ is discontinuous

(2) $\lim_{x \to c} f(x)$ exists

(3) $\lim_{x \to c} f(x) = f(c)$

If any of 3 conditions fail, then we say $f(x)$ is discontinuous at $x = c$.

Ex 1: Discuss continuity of $f(x) = \frac{2x}{x^2 - x}$.

When is $f(x)$ undefined? When denominator $= 0$.

$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, 1 \quad \Rightarrow \text{ We have discontinuities @ } x = 0, 1$$

But what kind of discontinuity are they? Jump? Hole? VA?

Let's simplify the function

$$f(x) = \frac{2x}{x^2 - x} = \frac{2\cancel{x}}{\cancel{x}(x-1)} = \frac{2}{x-1}$$

What factor cancelled? $\underline{x}$ $\Rightarrow$ $x = 0$ Hole

What factor remains (in denominator)? $\underline{x-1}$ $\Rightarrow$ $x = 1$ VA

Ex 2: Discuss continuity of $f(x) = \frac{x^2 + 2x - 3}{x^2 + 5x - 6}$

When is $f(x)$ undefined? Denominator $= 0$.

$$x^2 + 5x - 6 = 0$$
$$x^2 - x + 6x - 6 = 0$$
$$x(x-1) + 6(x-1) = 0$$
$$(x+6)(x-1) = 0$$
$$x = -6, 1 \quad \Rightarrow \text{ Hence we have discontinuities @ } x = -6, 1$$

-6

$-1 + 6 = 5$

But what kind of discontinuity are they? Jump? Hole? VA?

But what kind of discontinuity are they? Jump? ~~Hole~~? ~~VA~~?

$$f(x) = \frac{x^2+2x-3}{(x+6)(x-1)}$$

Let's factor the numerator

$$x^2+2x-3$$
$$=(x+3)(x-1)$$

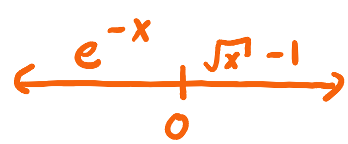
$$= \frac{(x+3)\cancel{(x-1)}}{(x+6)\cancel{(x-1)}}$$

What factor canceled? $\underline{x-1} \Rightarrow x=1$ Hole

What factor remains in denominator? $\underline{x+6} \Rightarrow x=-6$ VA

Ex 3: Discuss continuity of

$$f(x) = \begin{cases} e^{-x} & \text{if } x \le 0 \\ \sqrt{x}-1 & \text{if } x > 0 \end{cases}$$



To determine continuity, we need to check

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

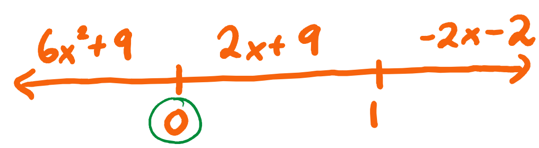
$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^{-x} = e^{-0} = e^0 = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sqrt{x}-1 = 0-1 = -1$$

$\neq \Rightarrow$ Discontinuity @ $x=0$ and moreso Jump @ $x=0$.

Ex 4: Discuss continuity of

$$f(x) = \begin{cases} 6x^2+9 & \text{if } x \le 0 \\ 2x+9 & \text{if } 0 < x < 1 \\ -2x-2 & \text{if } x \ge 1 \end{cases}$$



To determine continuity, we need to check

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

@ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 6x^2+9 = 6(0)^2+9 = 9$)) ✓ $\Rightarrow$ No discontinuity @ $x=0$

ⓐ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 6x^2+9 = 6(0)^2+9 = 9$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x+9 = 2(0)+9 = 9$

✓ $\Rightarrow$ No discontinuity @ $x=0$

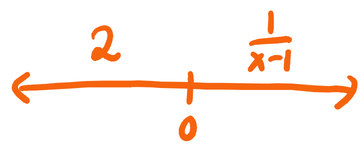
ⓑ $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x+9) = 2(1)+9 = 11$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (-2x-2) = -2(1)-2 = -4$

$\Rightarrow$ Jump @ $x=1$

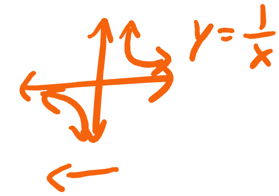
Ex 5: Discuss the continuity of

$$f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x>0 \\ 2 & \text{if } x \le 0 \end{cases}$$



To determine continuity, we need to check

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

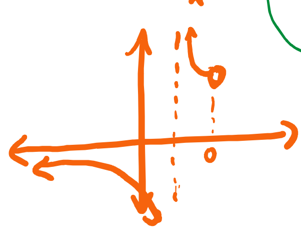


$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2 = 2$ $\Rightarrow$ Jump @ $x=0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \dfrac{1}{x-1} = \dfrac{1}{0-1} = \dfrac{1}{-1} = -1$

Check is $\dfrac{1}{x-1}$ when $x>0$ if it is continuous in that interval?

In addition VA @ $x=1$



$x \neq 1$ and $x>0$ $\Rightarrow$ 1 lives in $x>0$

$\Rightarrow$ VA @ $x=1$

This VA won't matter if $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x>1 \\ 2 & \text{if } x \le 1 \end{cases}$