Monday, January 26, 2026 12:36 PM

Lesson 6 Basic Rules of Differentiation
Derivatives of Sine and Cosine
Derivatives of the Natural Exponential Functions

2-weeks from Tuesday is Exam 1 8pm-9pm

TI 30 Xa

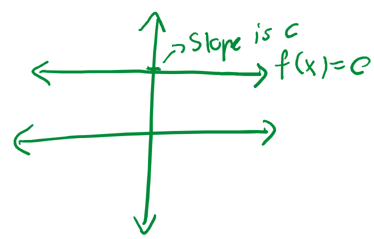
Sometime next week a calculator check

Basic Rules of Differentiation

① Constant Rule: For any constant c,

$$\frac{d}{dx}[c] = 0$$

Intuitively, this makes sense because the graph of $f(x)=c$ is a horizontal line.



Proof: Let's take derivative using the limit definition of derivatives.

Let $f(x) = c$

① $f(x+h) = c$

② $-f(x) = -c$

③ Add ①+② $= c - c = 0$

④ Divide by h: $\frac{0}{h} = 0$

⑤ $f'(x) = \lim_{h \to 0} 0 = 0 \Rightarrow \frac{d}{dx}[c] = 0$ Done!

② Power Rule: For any real number, n,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Ex 1: Find the derivative of $f(x) = x^2$

$n=2$

$$\frac{d}{dx}(x^2) = 2x^{2-1} = 2x^1 = 2x$$

Ex 2: Find the derivative of $f(x) = x^{-4}$

$n=-4$

Ex 2: Find the derivative of ...

$n = -4$

$$\frac{d}{dx}(x^{-4}) = -4x^{-4-1} = -4x^{-5} = -\frac{4}{x^5}$$

③ Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

④+⑤ Sum/Difference Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Ex 3: Find the derivative of $f(x) = x^5 + 5x^2$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[x^5 + 5x^2]$$

$$= \frac{d}{dx}[x^5] + \frac{d}{dx}[5x^2] \quad \text{(by Rule 4)}$$

$$= \frac{d}{dx}[x^5] + 5\frac{d}{dx}[x^2] \quad \text{(by Rule 3)}$$

$$= 5x^{5-1} + 5 \cdot 2x^{2-1} \quad \text{(by Power Rule)}$$

$$= 5x^4 + 10x$$

Ex 4: Find the derivative of

$$f(x) = \frac{3}{x^4} - 2x^2 + 6x + 7$$

$$\frac{1}{x^m} = x^{-m}$$

We want to rewrite $f(x)$ with no fractions

$$f(x) = 3x^{-4} - 2x^2 + 6x + 7$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[3x^{-4} - 2x^2 + 6x + 7]$$

$$= \frac{d}{dx}[3x^{-4}] - \frac{d}{dx}[2x^2] + \frac{d}{dx}[6x] + \frac{d}{dx}[7] \quad \text{(by Rule 4+5)}$$

$$= 3\frac{d}{dx}[x^{-4}] - 2\frac{d}{dx}[x^2] + 6\frac{d}{dx}[x^1] + \boxed{\frac{d}{dx}[7]} \quad \text{(by Rule 3)}$$

$= 3\frac{d}{dx}[x^{-4}] - 2\frac{d}{dx}[x^2] + 6\frac{d}{dx}[x^1] + \underbrace{\boxed{\frac{d}{dx}[7]}}_{=\,0\ \text{(by Rule 1)}}$ (by Rule 3)

$= 3(-4)x^{-4-1} - 2(2)x^{2-1} + 6\cdot 1 x^{1-1} + 0$

$= -12x^{-5} - 4x^1 + 6x^0$ $x^0 = 1$

$= -12x^{-5} - 4x + 6$

$= -\frac{12}{x^5} - 4x + 6$

Ex 5: Find the derivative of $f(x) = \sqrt{x^1} + \sqrt[3]{x^2}$

$\sqrt[2]{x^1} = x^{1/2}$

$\sqrt[3]{x^2} = (x^2)^{1/3} = x^{2/3}$

Rewrite to have numerical powers.

$f(x) = x^{1/2} + x^{2/3}$

$f'(x) = \frac{d}{dx}[x^{1/2} + x^{2/3}]$

$= \frac{d}{dx}[x^{1/2}] + \frac{d}{dx}[x^{2/3}]$

$= \frac{1}{2}x^{1/2-1} + \frac{2}{3}x^{2/3-1}$

$= \frac{1}{2}x^{-1/2} + \frac{2}{3}x^{-1/3}$

Derivatives of Sine and Cosine

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$\frac{d}{dx}(\cos(x)) = -\sin(x)$

Ex 6: Find y' of $y = 3\sin(x) - 4\cos(x)$

$y' = \frac{d}{dx}[3\sin(x)] - \frac{d}{dx}[4\cos(x)]$

$= 3\underbrace{\frac{d}{dx}[\sin(x)]}_{\cos(x)} - 4\underbrace{\frac{d}{dx}[\cos(x)]}_{-\sin(x)}$

$\cos(x)$ $\qquad$ $-\sin(x)$

$$= 3\cos(x) - 4(-\sin(x))$$

$$= 3\cos(x) + 4\sin(x)$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Ex 7: Find the x-value at which the derivative of $y = 10e^x$ is 1.

i.e. Solve $y' = 1$ for x.

$$y' = \frac{d}{dx}[10e^x] = 10\frac{d}{dx}[e^x] = 10e^x$$

I want $y' = 1$

$$10e^x = 1 \quad \leftarrow \text{solve for } x.$$

$$e^x = \frac{1}{10}$$

$$\ln(e^x) = \ln\left(\frac{1}{10}\right)$$

$$x = \ln\left(\frac{1}{10}\right)$$

Ex 8 (Challenge): Find the derivative of

$$f(x) = 2x^2 + \frac{3}{x^3} - \sqrt[4]{x^5} + 2\sin(x) - 5\cos(x) + \pi e^x$$

Rewrite the function

$$f(x) = 2x^2 + 3x^{-3} - x^{5/4} + 2\sin(x) - 5\cos(x) + \pi e^x$$

$$f'(x) = 2\frac{d}{dx}[x^2] + 3\frac{d}{dx}[x^{-3}] - \frac{d}{dx}[x^{5/4}] + 2\frac{d}{dx}[\sin(x)] - 5\frac{d}{dx}[\cos(x)]$$

$$+ \pi\frac{d}{dx}[e^x]$$

$$= 2\cdot 2x^{2-1} + 3(-3)x^{-3-1} - \frac{5}{4}x^{5/4-1} + 2\cos(x) - 5(-\sin(x)) + \pi e^x$$

$$= 2\cdot 2x^{2-1} + 3(-3)x^{-3-1} - \frac{5}{4}x^{5/4-1} + 2\cos(x) - 5(-\sin(x)) + \pi e^x$$

$$= 4x^1 - 9x^{-4} - \frac{5}{4}x^{1/4} + 2\cos(x) + 5\sin(x) + \pi e^x$$

$$= 4x - \frac{9}{x^4} - \frac{5}{4}\sqrt[4]{x} + 2\cos(x) + 5\sin(x) + \pi e^x$$