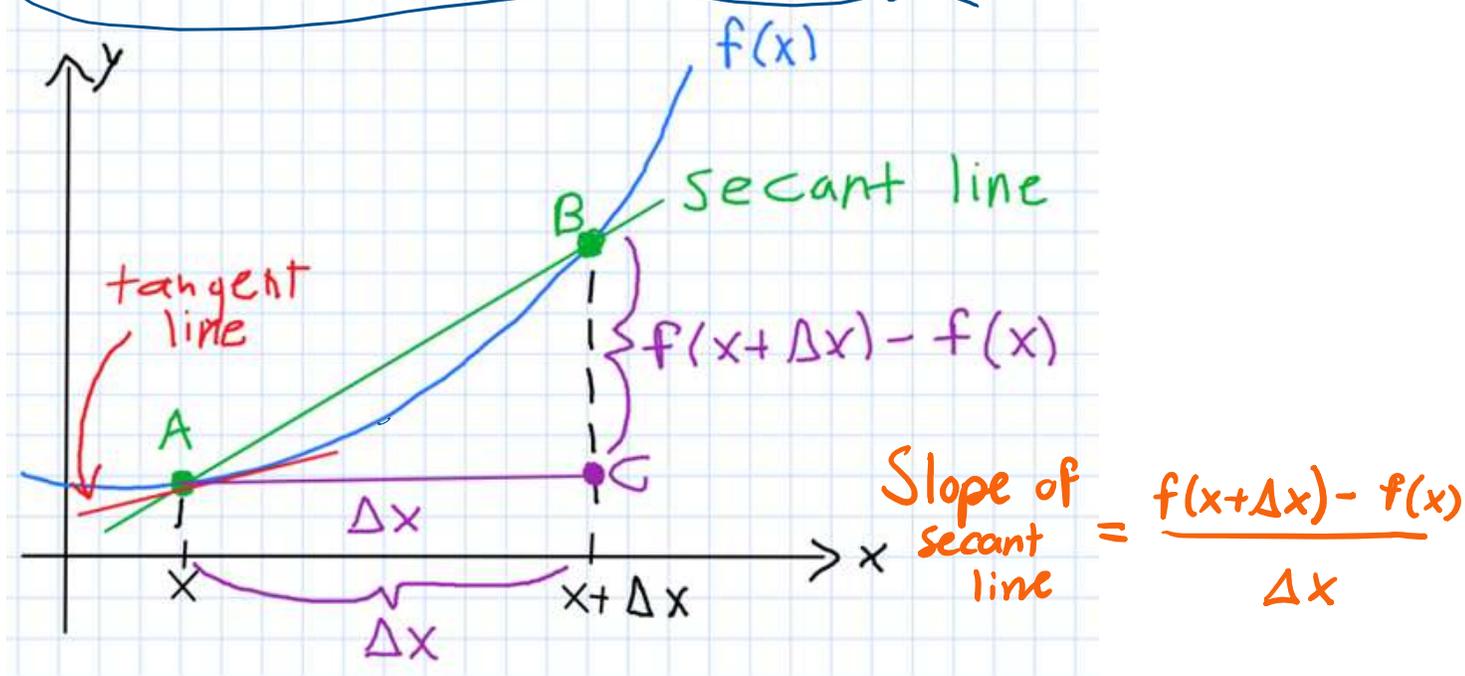


Lesson 7: Instantaneous Rates of Changes



This quantity is also known as the average rate of change.

Average rate of change approaches a quantity is called instantaneous rate of change.
i.e. it's the derivative of $f(x)$.

Ex 1: The initial population of a culture of bacteria is 1000. The population after t hours, $P(t)$, is given by

$$P(t) = 2t^2 + 8t + 1000$$

Ⓐ Find the number of bacteria present after 5 hrs.

$$P(5) = 2(5)^2 + 8(5) + 1000$$

$$\begin{aligned}
 P(5) &= 2(5)^2 + 8(5) + 1000 \\
 &= 2(25) + 40 + 1000 \\
 &= 50 + 40 + 1000 = 1090
 \end{aligned}$$

⑥ Find the rate of change of the population after 5 hrs.
 i.e. is derivative

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

What's $P'(5)$?

$$P'(t) = \frac{d}{dt}[2t^2 + 8t + 1000] = 2 \cdot 2t^{2-1} + 8 + 0 = 4t + 8$$

$$P'(5) = 4(5) + 8 = 20 + 8 = 28$$

Position & Velocity Functions

- Position Function $[s(t)]$ tells us how far away an object is
- Velocity function $[v(t)]$ tells us speed of an object with respect to direction.

To find velocity we take the derivative of the position.

$$v(t) = \frac{d}{dt}(s(t)) = \frac{ds}{dt} = s'(t)$$

Ex 2: An object is shot upward from the surface of ~~our~~ Earth.

The position function is

$$s(t) = -4.9t^2 + 98t$$

① Find $v(t)$.

$$\begin{aligned}
 v(t) = s'(t) &= \frac{d}{dt}[-4.9t^2 + 98t] = -4.9 \cdot 2t^{2-1} + 98 \\
 &= -9.8t + 98
 \end{aligned}$$

② Find $v(3)$

By ①, $v(t) = -9.8t + 98$

$$v(3) = -9.8(3) + 98 = 68.6$$

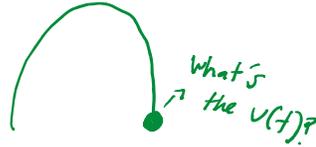
$$v(3) = -9.8(3) + 98 = 68.6$$

© What is the velocity of the object when it hits the ground?

i.e. Solve $s(t) = 0$ for t . Plug t into $v(t)$.

$$\begin{aligned} s(t) &= 0 \\ -4.9t^2 + 98t &= 0 \\ -4.9t + (t - 20) &= 0 \\ -4.9t &= 0 & | & t - 20 = 0 \\ t &= 0 & | & t = 20 \end{aligned}$$

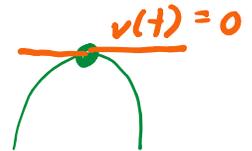
$$\left. \begin{aligned} v(t) &= -9.8t + 98 \\ v(20) &= -9.8(20) + 98 \\ &= -98 \end{aligned} \right\}$$



© When is the object at its highest point?

Solve $v(t) = 0$ for t .

$$\begin{aligned} v(t) &= -9.8t + 98 = 0 \\ 98 &= 9.8t \\ \frac{98}{9.8} &= \frac{9.8t}{9.8} \\ 10 &= t \end{aligned}$$



Ex 3: Let $C = 2\pi r$. What is the rate of change of C with respect to r ?

i.e. Find $\frac{dC}{dr}$. $\frac{dC}{dr} = \frac{d}{dr} [C]$

$$\frac{d}{dr} [C] = \frac{d}{dr} [2\pi r]$$

$$1 \cdot \frac{dC}{dr} = 2\pi \cdot 1 \frac{dr}{dr}$$

$$\frac{dC}{dr} = 2\pi$$

Ex 4: Let $p = 3q - 5$

© What is the rate of change of p $\left(\begin{array}{l} \text{© What is the rate of change of } q \text{ with} \\ \text{respect to } p? \end{array} \right)$

with respect to q ? $\frac{d}{dq}$

$$\frac{d}{dq}[p] = \frac{d}{dq}[3q-5]$$

$$1 \cdot \frac{dp}{dq} = 3 \frac{dq}{dq}$$

$$\frac{dp}{dq} = 3$$

respect to p ? $\frac{d}{dp}$

$$\frac{d}{dp}[p] = \frac{d}{dp}[3q-5]$$

$$1 \cdot \frac{dp}{dp} = 3 \cdot \frac{dq}{dp}$$

$$1 = 3 \frac{dq}{dp}$$

$$\frac{1}{3} = \frac{dq}{dp}$$