# Lesson 8: Product Rule

Product Rule says the derivative of $h(x) = u(x)\,v(x)$ is

$$\frac{d}{dx}[h(x)] = \frac{d}{dx}[u(x)v(x)]$$

$$= \frac{d}{dx}[u(x)]\,v(x) + u(x)\,\frac{d}{dx}[v(x)]$$

$$= u'(x)\,v(x) + u(x)\,v'(x)$$

**Ex 1:** Given $h(x) = 2x^3 e^x$. Compute $h'(x)$

Let $u(x) = 2x^3$, $v(x) = e^x$

$u'(x) = 6x^2$, $v'(x) = e^x$

By product rule,

$$h'(x) = u'(x)\,v(x) + u(x)\,v'(x)$$

$$= 6x^2 e^x + 2x^3 e^x$$

$$= (6x^2 + 2x^3)e^x$$

**Ex 2:** Given $h(x) = x^2 \sin(x)$. Compute $h'(\pi/6)$

Let $u(x) = x^2$, $v(x) = \sin(x)$

$u'(x) = 2x$, $v'(x) = \cos(x)$

By product rule,

$$h'(x) = u'(x)\,v(x) + u(x)\,v'(x)$$

$$= 2x\sin(x) + x^2\cos(x)$$

$$h'\left(\frac{\pi}{6}\right) = 2\frac{\pi}{6}\sin\left(\frac{\pi}{6}\right) + \left(\frac{\pi}{6}\right)^2\cos\left(\frac{\pi}{6}\right)$$

$$= \frac{\pi}{3}\cdot\frac{1}{2} + \frac{\pi^2}{36}\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{11}{3} \cdot \frac{1}{2} + \frac{11}{36} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{11}{6} + \frac{11^2\sqrt{3}}{72}$$

**Ex 3:** Given $h(x) = \sqrt{x}(2x^2+4)$. Compute $h'(x)$.

**Method 1: Use product rule**

Let $u(x) = x^{1/2}$ $\quad v(x) = 2x^2+4$

$u'(x) = \frac{1}{2}x^{-1/2}$ $\quad v'(x) = 4x$

By product rule,

$$h'(x) = u'(x)v(x) + u(x)v'(x)$$

$$= \frac{1}{2}x^{-1/2}(2x^2+4) + x^{1/2} \cdot 4x$$

$$= 1x^{3/2} + 2x^{-1/2} + 4x^{3/2}$$

$$= 5x^{3/2} + 2x^{-1/2}$$

**Method 2: Expand $h(x)$**

$$h(x) = x^{1/2}(2x^2+4)$$

$$= 2x^{5/2} + 4x^{1/2}$$

By Power Rule,

$$h'(x) = 2 \cdot \frac{5}{2}x^{3/2} + 4 \cdot \frac{1}{2}x^{-1/2}$$

$$= 5x^{3/2} + 2x^{-1/2}$$

**Moral:** Just b/c there is a product doesn't mean you need to use product rule.

**Ex 4:** Given $h(x) = (x^2+5x)(-3x^5+6)$. Find $h'(x)$.

Expand $h(x)$

| | $-3x^5$ | $6$ |
|---|---|---|
| $x^2$ | $-3x^7$ | $6x^2$ |
| $5x$ | $-15x^6$ | $30x$ |

$$h(x) = -3x^7 - 15x^6 + 6x^2 + 30x$$

By Power Rule,

$$h'(x) = -21x^6 - 90x^5 + 12x + 30$$

**Ex 5:** Given $h(x) = (x^3+x+1)(x^2+1)$. Find $h'(x)$.

| | $x^2$ | $1$ |
|---|---|---|
| $x^3$ | $x^5$ | $x^3$ |
| $x$ | $x^3$ | $x$ |
| $1$ | $x^2$ | $1$ |

$$h(x) = x^5 + x^3 + x^2 + x^3 + x + 1$$

$$= x^5 + 2x^3 + x^2 + x + 1$$

By Power Rule,

$$h'(x) = 5x^4 + 6x^2 + 2x + 1$$

**Ex 6:** Find x-values at which $y = 4x^6 e^x$ has a horizontal tangent.

Tangent $\Rightarrow f'(x)$ } $f'(x)$ is a slope } $f'(x) = 0$

Horizontal $\longleftrightarrow$ } Horizontal lines have slope $= 0$ }

First find $y'$.

Let $u(x) = 4x^6$ $\quad v(x) = e^x$

$u'(x) = 24x^5$ $\quad v'(x) = e^x$

By Product Rule

$y' = u'(x)v(x) + u(x)v'(x)$

$= 24x^5 e^x + 4x^6 e^x$

$= e^x(24x^5 + 4x^6)$

Solve $y' = 0$.

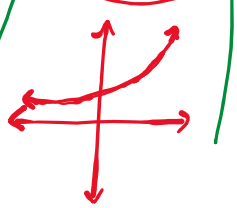
$e^x(24x^5 + 4x^6) = 0$

$e^x(4x^5)(6 + x) = 0$

$e^x = 0$ | $4x^5 = 0$ | $6 + x = 0$

$x^5 = 0$ | $x = -6$

$x = 0$



NEVER!

Ans: $x = -6, 0$

**Ex 7:** Let $h(x) = e^x \cos(x)$. Find $h'(x)$.

$u(x) = e^x$ $\quad v(x) = \cos(x)$

$u'(x) = e^x$ $\quad v'(x) = -\sin(x)$

By product rule,

$h'(x) = u'(x)v(x) + u(x)v'(x)$

$= e^x \cos(x) - e^x \sin(x)$

$= e^x[\cos(x) - \sin(x)]$

**Ex 8:** Let $h(x) = (x^2 + 2x + 1)(\cos x - \sin x + e^x)$. Compute

Let $u(x) = x^2 + 2x + 1$ $\quad v(x) = \cos(x) - \sin(x) + e^x$

$u'(x) = 2x + 2$ $\quad v'(x) = -\sin(x) - \cos(x) + e^x$

Let $u(x)$ [illegible]

$u'(x) = 2x + 2 \qquad v'(x) = -\sin(x) - \cos(x) + e^x$

By product rule,

$h'(x) = u'(x)\,v(x) + u(x)\,v'(x)$

$= (2x+2)(\cos x - \sin x + e^x) + (x^2 + 2x + 1)(-\sin(x) - \cos(x) + e^x)$