Monday, February 2, 2026 1:16 PM

Calculator Check on 2/4 TI 30Xa }

12 questions
Multiple Choice
Lessons 2-10
8pm-9pm Tuesday 2/10

Lesson 9: Quotient Rule; Derivatives of other Trig Functions

Quotient Rule:

Quotient Rule says the derivative of $h(x) = \frac{u(x)}{v(x)}$ is

$$\frac{d}{dx}[h(x)] = \frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

Ex 1: Let $h(x) = \frac{1}{x^2}$. Find $h'(x)$

Method 1: Power Rule

Rewrite $h(x)$

$$h(x) = x^{-2}$$

$$h'(x) = -2x^{-2-1}$$

$$= -2x^{-3}$$

$$= -\frac{2}{x^3}$$

Method 2: Quotient Rule

Let $u(x) = 1$ $v(x) = x^2$

$u'(x) = 0$ $v'(x) = 2x$

By quotient rule,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

$$= \frac{0 \cdot x^2 - 1 \cdot (2x)}{(x^2)^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}$$

Ex 2: Let $h(x) = \frac{x^2+1}{x^3-3x}$. Find $h'(x)$.

Let $u(x) = x^2+1$ $v(x) = x^3-3x$

$u'(x) = 2x$ $v'(x) = 3x^2-3$

By quotient rule,

$h'(x) = u'(x)v(x) - u(x)v'(x)$

By quotient rule,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

$$= \frac{2x(x^3-3x) - \underbrace{(x^2+1)(3x^2-3)}}{(x^3-3x)^2}$$

	x^2	1
$3x^2$	$3x^4$	$\cancel{3x^2}$
-3	$\cancel{-3x^2}$	-3

$$= \frac{2x(x^3-3x) - (3x^4-3)}{(x^3-3x)^2}$$

$$= \frac{2x^4-6x^2-3x^4+3}{(x^3-3x)^2}$$

$$= \frac{-x^4-6x^2+3}{(x^3-3x)^2}$$

Ex 3: Let $h(x) = \dfrac{\sin(x)}{x+\sin(x)}$. Find $h'(x)$.

Let $u(x) = \sin(x)$ $\qquad v(x) = x + \sin(x)$

$u'(x) = \cos(x)$ $\qquad v'(x) = 1 + \cos(x)$

By quotient rule,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

$$= \frac{\cos(x)[x+\sin(x)] - \sin(x)[1+\cos(x)]}{(x+\sin(x))^2}$$

$$= \frac{x\cos(x) + \cancel{\cos(x)\sin(x)} - \sin(x) - \cancel{\sin(x)\cos(x)}}{(x+\sin(x))^2}$$

$$= \frac{x\cos(x) - \sin(x)}{(x+\sin(x))^2}$$

$$= \frac{x\cos(x) - \sin(x)}{(x+\sin(x))^2}$$

Ex 4: Let $h(x) = \frac{x^2+2x+\pi}{e^x}$ Find $h'(x)$

Let $u(x) = x^2+2x+\pi$ $\quad v(x) = e^x$

$u'(x) = 2x+2$ $\quad v'(x) = e^x$

By quotient rule,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

$$= \frac{(2x+2)e^x - (x^2+2x+\pi)e^x}{(e^x)^2}$$

$$= \frac{(\cancel{2x}+2-x^2-\cancel{2x}-\pi)\cancel{e^x}}{(e^x)^{\cancel{2}}}$$

$$= \frac{(-x^2+2-\pi)}{e^x}$$

Derivatives of Other Trig Functions

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ $\quad \cot(x) = \frac{\cos(x)}{\sin(x)}$

$\sec(x) = \frac{1}{\cos(x)}$ $\quad \csc(x) = \frac{1}{\sin(x)}$

Ex 5: Let $h(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$. Find $h'(x)$

Let $u(x) = \sin(x)$ $\quad v(x) = \cos(x)$

$u'(x) = \cos(x)$ $\quad v'(x) = -\sin(x)$

By quotient rule,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

$$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$\boxed{\cos^2(x) + \sin^2(x) = 1 \text{ Always}}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x) \quad \Rightarrow \quad \frac{d}{dx}[\tan(x)] = \sec^2(x)$$

$$\left.\begin{aligned} \frac{d}{dx}[\cot(x)] &= -\csc^2(x) \\ \frac{d}{dx}[\sec(x)] &= \sec(x)\tan(x) \\ \frac{d}{dx}[\csc(x)] &= -\csc(x)\cot(x) \end{aligned}\right\} \text{Can be derived using quotient rule}$$

Ex 6: Given $h(x) = 3\sin(x)\tan(x)$. Compute $h'(x)$.

Let $u(x) = 3\sin(x)$ $\qquad v(x) = \tan(x)$

$u'(x) = 3\cos(x)$ $\qquad v'(x) = \sec^2(x)$

By product rule,

$$h'(x) = u'(x)v(x) + u(x)v'(x)$$

$$= 3\cos(x)\tan(x) + 3\sin(x)\sec^2(x)$$

$$= 3\cancel{\cos(x)}\frac{\sin(x)}{\cancel{\cos(x)}} + 3\sin(x)\sec^2(x)$$

$$= 3\sin(x) + 3\sin(x)\sec^2(x)$$

Ex 7: Given $h(x) = \dfrac{\tan(x)}{e^x + \sec(x)}$. Find $h'(x)$.

Let $u(x) = \tan(x)$ $\qquad v(x) = e^x + \sec(x)$

$e^{\hat{}} + \sec(x)$

Let $u(x) = \tan(x)$ $\quad v(x) = e^x + \sec(x)$

$u'(x) = \sec^2(x)$ $\quad v'(x) = e^x + \sec(x)\tan(x)$

By quotient rule,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

$$= \frac{\sec^2(x)\left[e^x + \sec(x)\right] - \tan(x)\left[e^x + \sec(x)\tan(x)\right]}{(e^x + \sec(x))^2}$$

$$= \frac{e^x \sec^2(x) + \sec^3(x) - \tan(x)e^x - \sec(x)\tan^2(x)}{(e^x + \sec(x))^2}$$