Exam 3 Formula Sheet

First-Order Linear Differential Equations

Any solution $y=f(x)$ of a first-order linear differential equation in standard form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Must satisfy $y \cdot u(x) = \int u(x)Q(x)dx$ where $u(x) = e^{\int P(x)dx}$.

Geometric Series:

The geometric series $\sum_{n=0}^{\infty} ar^n$ with a common ratio r converges if $|r| < 1$ with the sum

$$S = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Power Series/Maclaurin Series:

$$\frac{1}{1-\mathrm{x}} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{x}^{\mathrm{n}} = 1 + x + \mathrm{x}^2 + \mathrm{x}^3 + \cdots, |\mathrm{x}| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\ln(x+1) = \sum_{n=1}^{\infty} (-1)^{n-1} n = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$