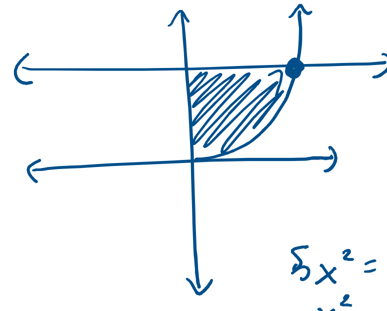
Shell Problems

① Which of the following integrals could calculate the solid generated by revolving the region bounded by the curves $y = 5x^2$, $x = 0$, and $y = 5$ about the y-axis using cylindrical shells.

$$V = 2\pi \int_a^b x\,(\text{Top} - \text{Bottom})\,dx$$

$$= 2\pi \int_0^1 x\,(5 - 5x^2)\,dx$$

$$= 10\pi \int_0^1 (x - x^3)\,dx$$



$5x^2 = 5$
$x^2 = 1$
$x = 1$

② Calculate the volume of the solid obtained by revolving the region enclosed by the curves $x = 6y - y^2$ and $x = 0$ about the x-axis.

$$V = 2\pi \int_a^b y\,(\text{Right} - \text{Left})\,dy$$

$6y - y^2 = 0$
$y(6 - y) = 0$
$y = 0, 6$

$$= 2\pi \int_0^6 y\,(6y - y^2 - 0)\,dy$$

$$= 2\pi \int_0^6 6y^2 - y^3\,dy$$

$$= 2\pi \left(\frac{6y^3}{3} - \frac{y^4}{4}\right)\Big]_0^6$$

$$= 2\pi \left(2y^3 - \frac{y^4}{4}\right)\Big]_0^6 = 2\pi\left(2\cdot 6^3 - \frac{6^4}{4}\right)$$

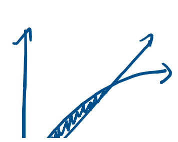
Test pt: $x = 1$

$x = 6y - y^2 \rightarrow x = 5$
$x = 0 \rightarrow 0$

③ Which of the following integrals would calculate the volume of the solid generated by revolving the region bounded by the curves $y = 2\sqrt{x}$ and $y = x$ about the line $x = 7$ using cylindrical shells.

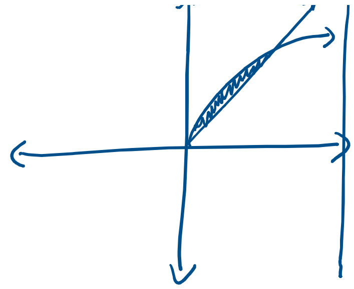
$\downarrow$
dx

Right $\Rightarrow 7 - x$



$= 2\pi \int \ldots \; (\text{Top} - \text{Bottom})\,dx$

Right $\Rightarrow 7-x$



$$V = 2\pi\int_a^b (7-x)(\text{Top} - \text{Bottom})\,dx$$

$2\sqrt{x} = x$

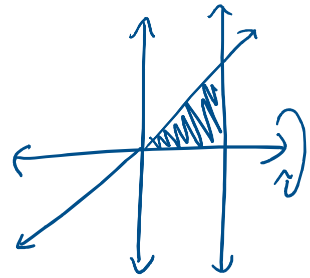
$4x = x^2$

$0 = x^2 - 4x$

$x = 0, 4$

$$= 2\pi\int_0^4 (7-x)(2\sqrt{x} - x)\,dx$$

④ Find the volume of the solid generated by revolving the region enclosed by the curves $y = \frac{x}{4}$, $x = 24$, and $y = 0$ about the x-axis



$$V = \pi\int_0^{24}\left(\frac{x}{4}\right)^2 dx$$

$$= \pi\int_0^{24} \frac{x^2}{16}\,dx$$

$$= \frac{\pi}{16}\cdot\frac{x^3}{3}\Big]_0^{24} = \frac{\pi \cdot 24^3}{16 \cdot 3} = 288\pi$$

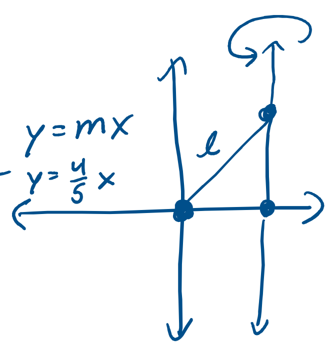
⑤ Find the volume obtained by revolving the region in the first quadrant enclosed by a triangle with vertices at (0,0), (5,0), and (5,4) about the line $x = 5$.

$\Downarrow$

Washer dy Shell dx

$y = mx$

$x = \frac{5}{4}y \leftarrow y = \frac{4}{5}x$



$$V = \pi\int_0^4 \left(\frac{5}{4}y - 5\right)^2 dy \qquad V = 2\pi\int_0^5 (5-x)\left(\frac{4}{5}x\right)dx$$

$u = \frac{5}{4}y - 5$

$du = \frac{5}{4}dy$

$\frac{4}{5}du = dy$

$$\pi\int \frac{4}{5}u^2\,du$$

$$= \pi\cdot\frac{4}{5}\cdot\frac{u^3}{3} = \pi\cdot\frac{4}{5}\cdot\frac{\left(\frac{5}{4}y-5\right)^3}{3}\Bigg]_0^4 = \frac{4\pi}{15}\left(\frac{5}{4}(4)-5\right)^3 - \frac{4\pi}{15}\left(\frac{5}{4}(0)-5\right)^3$$

$$= \pi \cdot \frac{4}{5} \cdot \frac{u^3}{3} = \pi \cdot \frac{4}{5} \cdot \frac{\left(\frac{5}{4}y-5\right)}{3}\Bigg|_0 = \frac{4\pi}{15}\left(\frac{5}{4}(4)-5\right) - \frac{4\pi}{15}\left(\frac{5}{4}(0)-5\right)$$

$$= +\frac{4\pi}{\underset{3}{\cancel{15}}}(-5)^{\cancel{3}\,2} = \frac{100\pi}{3}$$

⑥ Which of the following integrals would calculate the volume of the solid generated by the region enclosed by the curves $y = 7x - x^2$ and $y = x$ about the y-axis using shells?

$$7x - x^2 = x$$
$$6x - x^2 = 0$$
$$x = 0, 6$$

$$V = 2\pi \int_a^b x\,(\text{Top} - \text{Bottom})\,dx$$

$$= 2\pi \int_0^6 x\left(7x - x^2 - x\right)dx$$

$$= 2\pi \int_0^6 x\left(6x - x^2\right)dx$$

$$= 2\pi \int_0^6 \left(6x^2 - x^3\right)dx$$

$f_x(x,y)$ if $f(x,y) = \ln(2x^6y^4)$

$$f_x(x,y) = \frac{1}{2x^6y^4}\cdot\frac{\partial}{\partial x}(2x^6y^4)$$

$$= \frac{y^4}{2x^6y^4}\,\frac{\partial}{\partial x}(2x^6)$$

$$= \frac{\cancel{y^4}}{\cancel{2}x^{6}\cancel{y^4}}\cdot\frac{\cancel{2}\cdot 6x^{\cancel{5}}}{1}$$

$$= \frac{6}{x}$$

Now f_y.

$$f_y(x,y) = \frac{1}{2x^6y^4}\cdot\frac{\partial}{\partial y}(2x^6y^4)$$

$$= \frac{1}{2x^6y^4}\cdot 2x^6\,\frac{\partial}{\partial y}(y^4)$$

$$= \frac{\cancel{2x^6}}{\cancel{2x^6}y^4}\cdot 4y^{\cancel{3}}$$

$$= \frac{4}{y}$$

$f_x(x,y)$ if $f(x,y) = \ln(2x^6y^4)$

$$f_x(x,y) = \frac{1}{2x^6y^4} \cdot \frac{\partial}{\partial x}(2x^6y^4)$$

$$= \frac{y^4}{2x^6y^4}\, \frac{\partial}{\partial x}(2x^6)$$

$$= \frac{\cancel{y^4}}{\cancel{2}x^{\cancel{6}}\cancel{y^4}} \cdot \frac{\cancel{2}\cdot 6\cancel{x^5}}{1}$$

$$= \frac{6}{x}$$

Now f_y.

$$f_y(x,y) = \frac{1}{2x^6y^4} \cdot \frac{\partial}{\partial y}(2x^6y^4)$$

$$= \frac{1}{2x^6y^4} \cdot 2x^6\, \frac{\partial}{\partial y}(y^4)$$

$$= \frac{\cancel{2x^6}}{\cancel{2x^6}y^4} \cdot 4\cancel{y^3}$$

$$= \frac{4}{y}$$