Using power series

$$\int \frac{5}{1-4x}\,dx$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} (x)^n$$

$$= \int \sum_{n=0}^{\infty} 5(4x)^n\,dx$$

$$= \int \sum_{n=0}^{\infty} 5\cdot 4^n x^n\,dx$$

$$= \sum_{n=0}^{\infty} 5\cdot 4^n \int x^n\,dx$$

$$= \sum_{n=0}^{\infty} 5\cdot 4^n \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{2+x^2}\,dx = \frac{1}{2}\int \frac{1}{1-(-x^2/2)}\,dx$$

$$= \frac{1}{2}\int \sum_{n=0}^{\infty} \left(-\frac{x^2}{2}\right)^n dx$$

$$= \frac{1}{2}\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^n}\,dx$$

$$= \frac{1}{2}\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}\int x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}\cdot \frac{x^{2n+1}}{(2n+1)} + C$$

$$2\cdot \underbrace{2\cdot \ldots \cdot 2}_{n}$$

Maclaurin

Evaluate $\ln(1.21)$ using the first 3 terms

$$\ln(x+1) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = \boxed{x - \frac{x^2}{2} + \frac{x^3}{3}} - \frac{x^4}{4} + \dots$$

$$\ln(1.21) = \ln(0.21+1) = 0.21 - \frac{(0.21)^2}{2} + \frac{(0.21)^3}{3}$$

$$\int^{0.2} \sin(3x^3)\,dx \quad \text{using the first three terms} \qquad = 3x$$

$\ln(0.3) = -0.342t$

$t = \dfrac{\ln(0.3)}{-0.342}$

$|y| = e^{-1/x + c}$

$\pm y = e^{c} e^{-1/x}$

$u(t) =$

$+ \dfrac{3^5 \cdot x^{16}}{16 \cdot 120}$

$2)^4 + \dfrac{3^5 (0.2)^{16}}{16 \cdot 120}$

$\dfrac{dv}{dt} = 2V^{2/3}$

$V(?) = 1331$

$V(0) = 0$

$dv = 2v^{2/3} dt$

$\int v^{-2/3} dv = \int 2 dt$

$3v^{1/3} = 2t + c$

$V(0) = 0$

$0 = 0 + c$

$3V^{1/3} = 2t$

$3(1331)^{1/3} = 2t$

$\frac{3}{2}(1331)^{1/3} = t$

$y \cdot u(t$

$y e^{-}$

$y e^{-}$

$y e^{-}$

$y =$

$=$

$y' = ky \Rightarrow y = Ce^{kt}$

$y' = -0.342y \Longrightarrow y = Ce^{-0.342t}$

Initial $y(0) = 35$ grams $\quad y = 35e^{-0.342t}$

What time $t$ in hrs does 30% of the mass?

$$0.3(\cancel{35}) = \cancel{35}e^{-0.342t}$$

$$\ln(0.3) = -0.342t$$

$$t = \frac{\ln(0.3)}{-0.342}$$

$\frac{x^4}{4} + \ldots$

$\frac{(0.21)^3}{3}$

$= \frac{3x^4}{4} - \frac{27x^{10}}{6 \cdot 10} + \frac{3^5 \cdot x^{16}}{16 \cdot 120}$

Sepa

$\frac{dy}{dx} = \frac{y}{x^2}$

$x^2 dy = y$

$\int \frac{1}{y} dy = \int$

$\ln|y| =$

$|y| = e$

$\pm y = e$

$\frac{dv}{dt} = 2V^{2/3}$ ; $V(0) = 0$

$0 = 0 +$

$V(3) = 1331$

## Separable

$$\frac{dy}{dx} = \frac{y}{x^2}$$

$$x^2\,dy = y\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{x^2}\,dx$$

$$\ln|y| = -\frac{1}{x} + C$$

$$|y| = e^{-1/x + C}$$

$$\pm y = e^{C} e^{-1/x}$$

$$y = \pm e^{C} e^{-1/x}$$

$$y = Ce^{-1/x}$$

$$v(0) = 0$$

$$0 = 0 + C$$

## First-order

$$\frac{dy}{dt} - 4y = 32e$$

$$P(t) = -4 \quad Q(t$$

$$u(t) = \exp[\int P($$

$$= \exp[\int -$$

$$= \exp[-$$

$$y \cdot u(t) = \int u($$

$$\ln(x+1) = \sum_{n=1} (-1)^{n-1} \frac{x^n}{n} = \boxed{x - \frac{x^2}{2} + \frac{x^3}{3}} - \frac{x^4}{4} + \ldots$$

$$\ln(1.21) = \ln(0.21+1) = 0.21 - \frac{(0.21)^2}{2} + \frac{(0.21)^3}{3}$$

... in his does ...

of the mass?

$0.3(\cancel{35}) = \cancel{35}e^{-0}$

$\ln(0.3) = -0.34$

$t = \frac{\ln(0.3)}{-0.342}$

$\int_0^{0.2} \sin(3x^3)\,dx$ using the first three terms.

$$\sin(x) = \sum_{n=0}^{\infty} \ldots\ldots = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$$

$$\int \sin(3x^3) = (3x^3) - \frac{(3x^3)^3}{3!} + \frac{(3x^3)^5}{5!}$$

$$= \int 3x^3 - \frac{27x^9}{6} + \frac{3^5 \cdot x^{15}}{120}\,dx$$

$$= \frac{3x^4}{4} - \frac{27x^{10}}{6 \cdot 10} + \frac{3^5 \cdot x^{16}}{16 \cdot 120}$$

$$\frac{3(0.2)^4}{4} - \frac{27(0.2)^4}{6 \cdot 10} + \frac{3^5(0.2)^{16}}{16 \cdot 120}$$

$\frac{dv}{dt} =$

$V($

$V(0)$

$dv =$

$\int v^{-2/3}$

$3v^{1/3}$

$$y = Ce^{-1/x}$$

First-order

$$\frac{dy}{dt} - 4y = 32e^{5t}$$

$$P(t) = -4 \qquad Q(t) = 32e^{5t}$$

$$u(t) = \exp\left[\int P(t)\,dt\right]$$

$$= \exp\left[\int -4\,dt\right]$$

$$= \exp[-4t] = e^{-4t}$$

$$y \cdot u(t) = \int u(t)\,Q(t)\,dt$$

$$ye^{-4t} = \int e^{-4t}\,32e^{5t}\,dt$$

$$ye^{-4t} = \int 32e^{t}\,dt$$

$$ye^{-4t} = 32e^{t} + C$$

$$y = e^{4t}\left(32e^{t} + C\right)$$

$$= 32e^{5t} + Ce^{4t}$$