Exam 1 Info

Wednesday @ 6:30-7:30pm in ES 2107
Scientific Calculator Only!
Lessons 1-7
Practice Exam sent!

12 questions
Multiple Choice
PUID necessary!

Lesson 7: Integration by Parts I

Recall the Product Rule,

$$(uv)' = u'v + uv'$$

what if we integrate both sides with respect to x

$$\int (uv)'dx = \int (u'v + uv')dx$$

$$uv = \int (u'v)dx + \int (uv')dx$$

Remember $u' = \frac{du}{dx}$ and $v' = \frac{dv}{dx}$. So

$$uv = \int \frac{du}{\cancel{dx}} \cdot v\cancel{dx} + \int u\frac{dv}{\cancel{dx}} \cdot \cancel{dx}$$

$$uv = \int vdu + \underbrace{\int udv}_{\text{Solve}}$$

$$uv - \int vdu = \int udv$$

Interesting enough is that for many cases an integral will have the form $\int udv$.

This technique of turning one integral into another is called Integration by Parts.

Formula: $\int udv = uv - \int vdu$

To use this technique:

- Choose u to be the one to differentiate
- Choose dv to be integrated

Remark: Sometimes picking u and dv can be tricking. There is an acronym that makes picking u easier. Think of it as a sort of order of operation for choosing u.

L – Logarithmic
A – Algebraic (i.e. polynomial)
T – Trigonometric
E – Exponential

Ex 1: Evaluate

ⓐ $\int x\ln(x)dx$ $\quad \dfrac{u=\ln(x)}{du=\frac{1}{x}dx} \quad \dfrac{dv=x\,dx}{v=\frac{x^2}{2}}$ $\quad uv-\int v\,du = \ln(x)\cdot\frac{x^2}{2}-\int\frac{x^2}{2}\cdot\frac{1}{x}dx$

$= \dfrac{x^2\ln(x)}{2}-\dfrac{1}{2}\int x\,dx = \dfrac{x^2\ln(x)}{2}-\dfrac{1}{2}\cdot\dfrac{x^2}{2}+C$

$= \dfrac{x^2\ln(x)}{2}-\dfrac{x^2}{4}+C$

ⓑ $\int x\cos x\,dx$ $\quad \dfrac{u=x}{du=dx} \quad \dfrac{dv=\cos(x)dx}{v=\sin(x)}$ $\quad uv-\int v\,du$

$= x\sin(x)-\int\sin(x)dx$

$= x\sin(x)-(-\cos(x))+C$

$= x\sin(x)+\cos(x)+C$

ⓒ $\int xe^{2x}dx$ $\quad \dfrac{u=x}{du=dx} \quad \dfrac{dv=e^{2x}dx}{v=\frac{1}{2}e^{2x}}$ $\quad uv-\int v\,du$

(c) $\int x e \quad dx$ $\quad \dfrac{}{du = dx} \quad \dfrac{}{v = \frac{1}{2}e^{2x}}$

$$= \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\cdot\frac{1}{2}e^{2x} + C$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

Ex 2: Evaluate

(a) $\displaystyle\int_0^{\pi/2} 8x\sin(x)dx \quad \frac{u = 8x}{du = 8dx} \quad \frac{dv = \sin(x)dx}{v = -\cos(x)} \quad uv - \int v\,du$

$$= -8x\cos(x) - \int(-\cos(x))\cdot 8\,dx$$

$$= -8x\cos(x) + 8\int\cos(x)dx$$

$$= \left(-8x\cos(x) + 8\sin(x)\right)\Big]_0^{\pi/2}$$

$$= \cancel{-\frac{8\pi}{2}\cos\left(\frac{\pi}{2}\right)}^{\,0} + 8\underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1} - \left(\cancel{-8\cdot 0\cos(0)}^{\,0} + \cancel{8\sin(0)}^{\,0}\right)$$

$$= 8$$

(b) $\displaystyle\int_1^e x\ln(\sqrt[3]{x})dx = \int_1^e x\cdot\frac{1}{3}\ln(x)dx = \int_1^e \frac{x}{3}\ln(x)dx$

$$\frac{u = \ln(x)}{du = \frac{1}{x}dx} \quad \frac{dv = x/3\,dx}{v = x^2/6} \quad uv - \int v\,du$$

$$= \ln(x)\cdot\frac{x^2}{6} - \int\frac{1}{x}\cdot\frac{x^2}{6}dx$$

$$= \frac{x^2\ln(x)}{6} - \int\frac{1}{6}\cdot x\,dx$$

$$\left(\frac{x^2\ln(x)}{6} - \frac{1}{}\frac{x^2}{}\right)\Big]^e$$

$$\overline{\quad 6 \quad} \qquad \int 6$$

$$= \left(\frac{x^2\ln(x)}{6} - \frac{1}{6}\cdot\frac{x^2}{2}\right)\Bigg]_1^e$$

$$= \frac{e^2\ln(e)}{6} - \frac{1}{12}\cdot e^2 - \left(\cancel{\frac{1^2\ln(1)}{6}}^{\,0} - \frac{1}{12}\cdot 1^2\right)$$

$$= \frac{2e^2}{2\cdot 6} - \frac{e^2}{12} + \frac{1}{12}$$

$$= \frac{e^2}{12} + \frac{1}{12}$$

Ⓒ $\displaystyle\int_1^e \frac{\ln(x)}{x^4}dx = \int_1^e x^{-4}\ln(x)\,dx$

$$\frac{u = \ln(x)}{du = \frac{1}{x}dx} \qquad \frac{dv = x^{-4}dx}{v = \frac{x^{-3}}{-3}} \qquad uv - \int v\,du$$

$$= \ln(x)\cdot\frac{x^{-3}}{-3} - \int \frac{x^{-3}}{-3}\cdot\frac{1}{x}\,dx$$

$$= \frac{-\ln(x)}{3x^3} + \int \frac{1}{3}x^{-4}\,dx$$

$$= \left(-\frac{\ln(x)}{3x^3} + \frac{1}{3}\cdot\frac{x^{-3}}{-3}\right)\Bigg]_1^e$$

$$= \left(-\frac{\ln(x)}{3x^3} - \frac{1}{9x^3}\right)\Bigg]_1^e$$

$$= \left(\frac{-\ln(e)}{3e^3} - \frac{1}{9e^3}\right) - \left(\cancel{\frac{-\ln(1)}{3(1)^3}}^{\,0} - \frac{1}{9\cdot 1^3}\right)$$

$$= -\frac{1\cdot 3}{3e^3\cdot 3} - \frac{1}{9e^3} + \frac{1}{9}$$

$$= \frac{-4}{9e^3} + \frac{1}{9}$$