

Lesson 8: Integration by Parts II

Recall from last time, we found the formula for Integration by Parts.

$$\int u dv = uv - \int v du$$

We were also given an acronym to help in choosing what u should be.

L - Logarithmic

A - Algebraic (like polynomials)

T - Trigonometric

E - Exponential

Example 1: Evaluate

$$\begin{aligned} \textcircled{a} \int (t-3)e^t dt & \quad \frac{u=t-3}{du=dt} \quad \frac{dv=e^t dt}{v=e^t} \quad uv - \int v du \\ & = (t-3)e^t - \int e^t dt = (t-3)e^t - e^t + C \\ & = (t-3-1)e^t + C = (t-4)e^t + C \end{aligned}$$

$$\begin{aligned} \textcircled{b} \int \ln x dx & \quad \frac{u=\ln x}{du=\frac{1}{x} dx} \quad \frac{dv=dx}{v=x} \quad uv - \int v du \\ & = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - \int dx \\ & = x \ln x - x + C \end{aligned}$$

Example 2: Evaluate

$$\begin{aligned} \textcircled{c} \int x^2 \sin x dx & \quad \frac{u=x^2}{du=2x dx} \quad \frac{dv=\sin x dx}{v=-\cos x} \quad uv - \int v du \\ & = -x^2 \cos x - \int (-\cos x) 2x dx \\ & = -x^2 \cos x + \int 2x \cos x dx \end{aligned}$$

Note to ^v integrate this we need to do another integration by parts

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$$\int 2x \cos x dx \quad \begin{array}{l} u=2x \quad dv=\cos x dx \\ du=2dx \quad v=\sin x \end{array} \quad uv - \int v du$$

$$= 2x \sin x - \int 2 \sin x dx = 2x \sin x - (-2 \cos x)$$

$$= 2x \sin x + 2 \cos x$$

$$= x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$\textcircled{b} \int 12x (\ln(4x))^2 dx \quad \begin{array}{l} u=(\ln(4x))^2 \quad dv=12x dx \\ du=2(\ln(4x)) \cdot \frac{1}{4x} \cdot 4 dx \quad v=\frac{12x^2}{2} \\ du=2(\ln(4x)) dx \quad v=6x^2 \end{array}$$

$$= 6x^2 (\ln(4x))^2 - \int 6x^2 \cdot \frac{2(\ln(4x))}{x} dx$$

$$= 6x^2 (\ln(4x))^2 - \int 12x \ln(4x) dx$$

Note to integrate this
we need to do another
integration by parts

$$\int 12x \ln(4x) dx \quad \begin{array}{l} u=\ln(4x) \quad dv=12x dx \\ du=\frac{1}{4x} \cdot 4 dx \quad v=\frac{12x^2}{2} \\ du=\frac{1}{x} dx \quad v=6x^2 \end{array}$$

$$= 6x^2 \ln(4x) - \int 6x^2 \cdot \frac{1}{x} dx$$

$$= 6x^2 \ln(4x) - \int 6x dx$$

$$= 6x^2 \ln(4x) - \frac{6x^2}{2} = 6x^2 \ln(4x) - 3x^2$$

$$\downarrow$$

$$= 6x^2 (\ln(4x))^2 - (6x^2 \ln(4x) - 3x^2) + C$$

$$= 6x^2 (\ln(4x))^2 - 6x^2 \ln(4x) + 3x^2 + C$$

Example 3: The velocity of a car over the time period $0 \leq t \leq 3$ is given by the function

$$v(t) = 50 + e^{-t/4} \text{ miles per hour}$$

where t is time in hours. What was the distance the car traveled in the first 30 minutes?

Note t is in hours, so 30 mins \Rightarrow 0.5 hrs

We need to compute

$$\int_0^{0.5} 50 + e^{-t/4} dt \quad \begin{array}{l} u = 50t \\ du = 50 dt \end{array} \quad \begin{array}{l} dv = e^{-t/4} dt \\ v = -4e^{-t/4} \end{array} \quad uv - \int v du$$

$$= 50t + (-4e^{-t/4}) \Big|_0^{0.5} - \int_0^{0.5} -4e^{-t/4} (50) dt$$

$$= -200te^{-t/4} \Big|_0^{0.5} + 200 \int_0^{0.5} e^{-t/4} dt$$

$$= -200te^{-t/4} \Big|_0^{0.5} + 200(-4)e^{-t/4} \Big|_0^{0.5}$$

$$= -200(0.5)e^{-0.5/4} - (-200(0)e^{-0/4}) - 800e^{-0.5/4}$$

$$- (-800e^{-0/4})$$

$$= -100e^{-1/8} - 0 - 800e^{-1/8} + 800$$

$$= -900e^{-1/8} + 800$$

$$\approx 5.75$$

Example 4: A model for the ability of a child to memorize information, measured on a scale from 1 to 100, is given by

$$M(t) = 1 + 3.4t + \ln(t)$$

when $2 \leq t \leq 8$ where t is the child's age in years. Find the child's average memorization ability between ages 3 and 6.

$$M_{\text{AVE}}(t) = \frac{1}{6-3} \int_3^6 (1 + 3.4t + \ln(t)) dt$$

$$= \frac{1}{3} \int_3^6 dt + \frac{3.4}{3} \int_3^6 t + \ln(t) dt$$

$$\begin{array}{l} u = \ln(t) \quad dv = t dt \\ du = \frac{1}{t} dt \quad v = \frac{t^2}{2} \end{array}$$

$$= \frac{1}{3} \left(\int_3^6 dt + \frac{3.4}{3} \left(\frac{t^2}{2} \ln(t) \right) \Big|_3^6 - \int_3^6 \frac{t^2}{2} \cdot \frac{1}{t} dt \right)$$

$$= \frac{1}{3} \left(\int_3^6 dt + \frac{3.4}{3} \left(\frac{t^2}{2} \ln(t) \right) \Big|_3^6 - \int_3^6 \frac{1}{2} t dt \right)$$

$$= \frac{1}{3} \left[t \right]_3^6 + \frac{3.4}{6} \left[t^2 \ln(t) \right]_3^6 - \frac{3.4}{6} \left[\frac{t^2}{2} \right]_3^6$$

$$= \frac{1}{3} (6) - \frac{1}{3} (3) + \frac{3.4}{6} (6)^2 \ln(6) - \frac{3.4}{6} (3)^2 \ln(3)$$

$$- \frac{3.4}{12} (6)^2 - \left(-\frac{3.4}{12} (3)^2 \right)$$

$$\approx 26.299$$